TWO PAPERS ON SEQUENTIAL BARGAINING

PAPER I: SEQUENTIAL BARGAINING MECHANISMS

PAPER II: BARGAINING WITH INCOMPLETE INFORMATION AN INFINITE-HORIZON MODEL WITH CONTINUOUS UNCERTAINTY

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by

PETER C. CRAMTON

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CENTER FOR RESEARCH ON ORGANIZATIONAL EFFICIENCY
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TWO PAPERS ON SEQUENTIAL BARGAINING*

by

Peter C. Cramton**

Paper I: Sequential Bargaining Mechanisms

1. Introduction

A fundamental problem in economics is determining how agreements are reached in situations where the parties have some market power. Of particular interest are questions of efficiency and distribution:

How efficient is the agreement?

How can efficiency be improved?

How are the gains from agreement divided among the parties?

Here I explore these questions in the context of bilateral monopoly, in which a buyer and a seller are bargaining over the price of an object.

Two features of my analysis, which are important in any bargaining setting, are information and impatience. The bargainers typically have private information about their preferences and will suffer some delay costs if agreement is postponed. Information asymmetries between

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bargainers will often lead to inefficiencies: the bargainers will be forced to delay agreement in order to communicate their preferences. Impatience will tend to encourage an early agreement and will make the parties' communication meaningful. Those with high delay costs will accept inferior terms of trade in order to conclude agreement early; whereas, patient bargainers will choose to wait for more appealing terms of trade.

Some authors have examined the bargaining problem in a static context, focusing solely on the role of incomplete information and ignoring the sequential aspects of bargaining. Myerson and Satterthwaite [1983] analyze bargaining as a direct revelation game. In this game, the players agree to a pair of outcome functions: one that maps the players' statements of their types into an expected payment from buyer to seller, and the other that maps the players' statements into a probability of trade. These outcome functions are chosen in such a way that truthful reporting is an equilibrium strategy for the players. An important feature of this game is that it is static; outcome functions are selected, the players report their true types, and then dice are rolled to determine the payment and whether or not trade occurs. In order to insure that the players have the proper incentives for truthful reporting, the game will end with positive probability in disagreement even when there are substantial gains from trade. Thus, in the event the randomization device calls for disagreement, the players may find themselves in a situation in which it is common knowledge that there are gains from trade.

Chatterjee and Samuelson [1983] analyze a strategic game in which both players simultaneously make offers and trade occurs at a price between the two offers if the seller's offer is less than that of the buyer. This game is closely related to the direct revelation game in that it is static. Moreover, it can be shown that for a particular class of examples the simultaneous-offer game implements the direct revelation game in which the outcome functions are chosen to maximize the players' ex ante utility. As in the direct revelation game, this game ends with positive probability in a state in which both bargainers know that gains are possible (since their respective reservation prices have been revealed), and yet they are forced to walk away from the bargaining table. Thus, the bargaining game implicitly assumes that the players are able to commit to walking away without trading, after it has been revealed that substantial gains from trade exist.

In situations where the bargainers are unable to make binding agreements, it is unrealistic to employ a bargaining mechanism that forces them to walk away from known positive gains from trade. Such mechanisms violate a broad interpretation of sequential rationality as discussed by Selten [1976] (in terms of subgame perfection) and later by Kreps and Wilson [1982], if one applies sequential rationality not only to the hypothesized game, but to the game form as well. In particular, one should restrict attention to mechanisms that satisfy sequential rationality: it must never be common knowledge that the mechanism induced at any point in time is dominated by an alternative mechanism.

When there is uncertainty about whether or not gains from trade exists, any static game will violate sequential rationality. The players must have time to learn through each other's actions whether gains are possible. In a sequential game, the players communicate their preferences by exhibiting their willingness to delay agreement. Bargainers that anticipate large gains from trade (low-cost sellers and high-valuation buyers) will be unwilling to delay agreement and so will propose attractive terms of trade that the other is likely to accept early in the bargaining process. On the other hand, high-cost sellers and low-valuation buyers will prefer to wait for better terms of trade. Static games must use a positive probability of disagreement to insure incentive compatibility, where the probability of disagreement increases as the gains from trade shrink. The advantage of delaying agreement rather than forbidding agreement is that mechansims can be constructed in which negotiations continue so long as each bargainer expects positive gains. Thus, the bargaining will not end in a state in which it is common knowledge that the players want to renege on their agreed upon outcome.

Two approaches can be taken in the analysis of perfect bargaining games. The first approach is to examine specific extensive-form (or strategic) games, which determine the set of actions available to the players over time. Intrinsic to any bargaining process is the notion of offers and replies: bargaining consists of a sequence of offers and decisions to accept or reject these offers. Who makes the offers, the time between offers, responses, and counter-offers, and the

possibilities for commitment are determined by the underlying communication technology present in the bargaining setting. This communication technology will imply, in part, a particular bargaining game in extensive form. Sobel and Takahashi [1983], Cramton [1983], and Fudenberg, Levine, and Tirole [1983] illustrate the analysis of particular extensive forms that are perfect bargaining games.

A second approach and the one adopted in this paper is to analyze a general direct revelation game, which maps the players' beliefs into bargaining outcomes. An important distinction between direct revelation games and strategic games is that the direct revelation game does not explicitly model the process of bargaining. The sequence of offers and replies that eventually leads to an outcome is not studied in the direct revelation game as it is in strategic games. However, embedded within each sequential bargaining mechanism is a particular form of learning behavior, which can be analyzed. In addition, much can be learned about how information and impatience influence the efficiency of the bargaining outcome and the allocation of gains between players. Thus, even though bargainers will not play direct revelation games in practice, their analysis is a useful tool to determine how well the bargainers can hope to do by adopting an appropriate strategic game.

The difference between the static direct revelation game analyzed by Myerson and Satterthwaite [1983] and the sequential direct revelation game considered here is that in the sequential game the outcome functions not only determine the probability and terms of trade, but also dictate when trade is to take place. In the static game, trade may

only occur at time zero; whereas, in the sequential game trade may occur at different times depending on the players' reports of their private information. Thus, by analyzing sequential bargaining mechanisms one is able to infer what the players' learning process is over time. Furthermore, by analyzing mechanisms that are sequentially rational, one can study what bargaining outcomes are possible when the bargainers are unable to make binding agreements.

This introductory paper considers the simplest type of sequential bargaining games in which the players' time preferences are described by known and fixed discount rates. I begin by characterizing the class of perfect bargaining mechanisms, which satisfy the desirable properties of incentive compatibility (each player reports his type truthfully), individual rationality (every potential player wishes to play the game), and sequential rationality (it is never common knowledge that the mechanism induced over time is dominated by an alternative mechanism). It is shown that ex post efficiency is unobtainable by any incentive-compatible and individually-rational mechanism when the bargainers are uncertain about whether or not they should trade immediately. I conclude by finding those mechanisms that maximize the players' ex ante utility, and show that such mechanisms violate sequential rationality. Thus, the bargainers would be better off ex ante if they could commit to a mechanism before they knew their private information. In terms of their ex ante payoffs, if the seller's delay costs are higher than those of the buyer, then the bargainers are better off adopting a sequential bargaining game rather than a static mechanism; however, when the

buyer's delay costs are higher, then a static mechanism is optimal.

The methodology of this paper is based on Myerson and Satterthwaite [1983]. I have freely borrowed from their insightful work in much of my analysis. Complete proofs to each proposition, even though many are only slightly different from the proofs found in Myerson and Satterthwaite, are given as an aid to the reader.

2. Formulation

Two parties, a buyer and a seller, are bargaining over the price of an object which can be produced by the seller at a cost s and is worth b to the buyer.[1/] The seller's cost s and the buyer's valuation b are also referred to as their reservation prices, since they represent respectively the minimum and maximum price at which each would agree to trade. Both the buyer and the seller have costs of delaying the bargaining process. Specifically, the value of the object is discounted in the future according to the positive discount rates ρ for the seller and σ for the buyer. Thus the payoffs, if the bargainers agree to trade at the discounted price x at time t, are $s - xe^{-\rho t}$ for the seller and $be^{-\sigma t} - x$ for the buyer. Should they fail to reach agreement both players' payoffs are zero. Implicit in this formulation is the assumption that the bargainers discount future money at the same rate, so at any time t the discounted payment by the buyer equals the discounted revenue to the seller. Without this assumption, it would be possible for the players to achieve an infinite payoff by having the player with the lower discount rate lend an

arbitrarily large amount of money to the other player.

The buyer, though aware of his own valuation b, does not know the seller's cost of production s, but assesses her cost to be distributed according to the distribution $F(s)$, with a positive density $f(s)$ on $[\underline{s},\bar{s}]$. Similarly, the seller knows her cost s, but can only assess the buyer's valuation to be distributed according to the distribution $G(b)$, with a positive density $g(b)$ on $[\underline{b},\bar{b}]$. Their discount rates and the distributions of the potential buyers and sellers are common knowledge. In addition, it is assumed that both the buyer and the seller are soley interested in maximizing their expected monetary gain.

To summarize, let $\langle F,G,\rho,\sigma\rangle$ be a sequential direct revelation game where

> F is the distribution of the seller's cost s on $[\underline{s},\bar{s}]$,
>
> G is the distribution of the buyer's valuation b on $[\underline{b},\bar{b}]$,
>
> ρ is the seller's discount rate for the object, and
>
> σ is the buyer's discount rate for the object.

In the revelation game, the player's actions consist of reports of their types, which are mapped into the bargaining outcome by the bargaining mechanism. Thus, the seller s reports that her cost is $s' \in [\underline{s},\bar{s}]$ and the buyer b reports that his valuation is $b' \in [\underline{b},\bar{b}]$. The revelation game is said to be <u>direct</u> is the equilibrium strategies of the players involve truthful reporting: $(s',b') = (s,b)$. The important role of direct revelation games stems from the fact that one can, without loss of generality, restrict attention to direct mechanisms.

For any Nash equilibrium of any bargaining game, there is an equivalent direct mechanism that always yields the same outcomes. This well known result is called the revelation principle. Given any mechanism M, which maps reports into outcomes, and a set of equilibrium strategies x, which maps true types into reported types, then the composition $\hat{M} = M \circ x$ is a direct mechanism that achieves the same outcomes as the mechanism M.

For the revelation game $\langle F,G,\rho,\sigma \rangle$, a sequential bargaining mechanism is the pair of outcome functions, $T(\cdot|\cdot,\cdot)$ and $x(\cdot,\cdot)$, where $T(t|s,b)$ is the probability distribution that the object will be transferred to the buyer at time t and $x(s,b)$ is the discounted expected payment from the buyer to the seller, given that the seller and buyer report the reservation prices s and b, respectively.

Typically, randomization of the outcomes over time is not necessary. Without randomization, the outcome function T can be replaced by the function $t(\cdot,\cdot)$, which determines the time of trade given the players' reports. A sequential bargaining mechanism, then, is the set of outcome functions $\langle t,x \rangle$ where $t(s,b)$ is the time of trade and $x(s,b)$ is the discounted expected payment, given that the seller reports s and the buyer reports b. Most bargaining mechanisms seen in practice require that the exchange of money and goods take place at the same time. Such a requirement is not restrictive in this model, because there is no benefit to be gained by exchanging money at a different time from the exchange of the good, since both players have identical time preferences for money. For reasns of tractability, I

will frequently restrict attention to the simplified mechanism $<t,x>$.

3. Perfect Bargaining Mechanisms

The weakest requirements one would wish to impose on the bargaining mechanism $<T,x>$ in the direct revelation game are

(1) individual rationality, that everyone wishes to play the game, and

(2) incentive compatibility, that the mechanism induces truth telling.

In addition, when the bargainers are unable to make binding commitments, one needs the further restriction of sequential rationality: it must never be common knowledge that the mechanism induced over time is dominated by an alternative mechanism. Bargaining schemes that satisfy incentive compatibility, individual rationality, and sequential rationality are called perfect bargaining mechanisms. The adjective "perfect" is adopted, because of the close relationship between perfect bargaining mechanisms in the direct revelation game and perfect (or sequential) equilibria in an infinite-horizon extensive-form game. It remains to be proven that a sequential bargaining mechanism is perfect if and only if it is a perfect equilibrium for some infinite-horizon extensive-form game. This issue will be addressed in future research.

In this section, I derive necessary and sufficient conditions for the sequential bargaining mechanism to be perfect. The incentive-compatibility and individual-rationality conditions were first established in Myerson and Satterthwaite [1983], and later extended to

the case of multiple buyers and sellers by Wilson [1982] and Gresik and Satterthwaite [1983]. It is important to realize that these properties are actually necessary and sufficient conditions for any Nash equilibrium of any bargaining game, since every Nash equilibrium induces a direct revelation mechanism as mentioned in section 2.

Incentive Compatibility

In order to define and determine the implications of incentive compatibility on the sequential bargaining mechanism $\langle T,x\rangle$, it is convenient to divide each player's expected payoff into two components as follows. Let

$$S(s) = \sum_{\underline{b}}^{\bar{b}} x(s,b)g(b)db \quad ; \quad P(s) = \sum_{\underline{b}}^{\bar{b}} \int_{0}^{\infty} e^{-\rho t} dT(t|s,b)g(b)db$$

$$B(b) = \sum_{\underline{s}}^{\bar{s}} x(s,b)f(s)ds \quad ; \quad Q(b) = \sum_{\underline{s}}^{\bar{s}} \sum_{0}^{\infty} e^{-\sigma t} dT(t|s,b)f(s)ds$$

where $S(s)$ is the discounted expected revenue and $P(s)$ is the discounted probability of agreement for seller s, and $B(b)$ is the discounted expected payment and $Q(b)$ is the discounted probability of agreement for buyer b. Thus the seller's and buyer's discounted expected payoffs are given by

$$U(s) = S(s) - sP(s) \qquad V(b) = bQ(b) - B(b) \quad .$$

Formally, the sequential bargaining mechanism $\langle T,x\rangle$ is incentive compatible if every type of player wants to report truthfully his type; that

is, for all s and s' in $[\underline{s},\bar{s}]$ and for all b and b' in $[\underline{b},\bar{b}]$:

$$U(s) > S(s') = sP(s') \quad ; \quad V(b) > bQ(b') - B(b') \quad .$$

<u>Lemma 1</u>: If the sequential bargaining mechanism $\langle\ ,x\rangle$ is incentive compatible. then the seller's expected payoff U is convex and decreasing, with derivative $dU/ds = -P$ almost everywhere on $[\underline{s},\bar{s}]$, her discounted probability of agreement P is decreasing. and

$$U(s) - U(\bar{s}) = \int_{s}^{\bar{s}} P(u)du \quad ; \quad S(s) - S(\bar{s}) = \int_{s}^{\bar{s}} - u\ dP(u) \quad . \qquad \text{(S)}$$

Similarly, the buyer's expected payoff V is convex and increasing, with derivative $dV/db = Q$ almost everywhere on $[\underline{b},\bar{b}]$, his discounted probability of agreement Q is increasing, and

$$V(b) - V(\underline{b}) = \int_{\underline{b}}^{b} Q(u)du \quad ; \quad B(b) - B(\underline{b}) = \int_{\underline{b}}^{b} u\ dQ(u) \qquad \text{(B)}$$

<u>Proof</u>: By definition, seller s achieves the payoff $U(s) = S(s) - sP(s)$. Alternatively, seller s can pretend to be seler s' in which case her payoff is $S(s') - sP(s')$. In the direct revelation game, the seller s must not want to pretend to be seller s', so we have $U(s) > S(s') - sP(s')$ for all s, $s' \in [\underline{s},\bar{s}]$, or

$$U(s) > U(s') - (s - s')P(s')$$

implying that U has a supporting hyperplane at s' with slope $-P(s') < 0$. Thus U is convex and decreasing with derivative

$dU/ds(s) = P(s)$ almost everywhere and P must be decreasing. Since P is monotone, it is differentiable almost everywhere and we have that $dS/ds = s(dP/ds)$, which yields (S). The proof for the buyer is identical. Q.E.D

Lemma 1 indicates the stringent requirements incentive compatibility imposes on the players' utilities. In particular, it suggests how one can construct an incentive-compatible payment schedule x, given a probability of agreement distribution T for which the seller's discounted probability of agreement $P(s)$ is decreasing in s and the buyer's discounted probability of agreement $Q(b)$ is increasing in b.

Lemma 2: Given the sequential bargaining mechanism $\langle T,x \rangle$ such that P is decreasing, Q is increasing, and S and B satisfy (S) and (B) of Lemma 1, then $\langle T,x \rangle$ is incentive compatible.

Proof: A mechanism is incentive compatible for the seller if for all $s, s' \in [\underline{s},\bar{s}]$,

$$S(s) - sP(s) \geq S(s') - sP(s') \quad .$$

Rearranging terms yields the following condition for incentive compatibility

$$s[P(s') - P(s)] + S(s) - S(s') \geq 0 \quad . \tag{S'}$$

From (S), we have

$$S(s) - S(s') = \int_{s}^{s'} - u \; dP(u)$$

and from the fundamental theorem of integral calculus[2]

$$s[P(s') - P(s)] = s\int_{s}^{s'} dP(u) \quad .$$

Adding the last two equations results in

$$s[P(s') - P(s)] + S(s) - S(s') = \int_{s}^{s'} (s - u)dP(u) \geq 0$$

where the inequality follows because the integrand $(s - u)dP(u)$ is nonnegative for all $s, u \in [\underline{s},\bar{s}]$, since P is decreasing. Hence, $\langle T,x\rangle$ satisfies the incentive-compatibility condition (S'). An identical argument follows for the buyer. Q.E.D.

<u>Individual Rationality</u>

The sequential bargaining mechanism $\langle T,x\rangle$ is individually rational if every type of player wants to play the game: for all s in $[\underline{s},\bar{s}]$ and b in $[\underline{b},\bar{b}]$,

$$U(s) \geq 0 \quad ; \quad V(b) \geq 0 \quad .$$

In light of the monotonicity of U and V proven in Lemma 1, any incentive-compatible mechanism $\langle T,x\rangle$ will satisfy individual rationality if the extreme high-cost seller and low-valuation buyer receive a nonnegative payoff; that is, an incentive-compatible mechanism $\langle T,x\rangle$ is individually rational if and only if $U(\bar{s}) \geq 0$ and $V(\underline{b}) \geq 0$.

The following lemma describes how one can check whether or not a sequential bargaining mechanism is individually rational. It is convenient to state the lemma in terms of the simplified bargaining mechanism <t,x> rather than <T,x>. Recall that for the sequential bargaining mechanism <t,x>, we have

$$S(s) \int_{\underline{b}}^{\bar{b}} x(s,b)g(b)db \quad ; \quad P(s) = \int_{\underline{b}}^{\bar{b}} e^{-\rho t(s,b)}g(b)db$$

$$B(b) = \int_{\underline{s}}^{\bar{s}} x(s,b)f(s)ds \quad ; \quad Q(b) = \int_{\underline{s}}^{\bar{s}} e^{-\sigma t(s,b)}f(s)ds \quad .$$

<u>Lemma 3</u>: If the sequential bargaining mechanism <t,x> is incentive compatible and individually rational, then

$$U(\bar{s}) + V(\underline{b}) = E\left\{\left[b - \frac{1 - G(b)}{g(b)}\right]e^{-\sigma t(s,b)} - \left[s + \frac{F(s)}{f(s)}\right]e^{-\rho t(s,b)}\right\} \geq 0 \quad , \tag{US}$$

where the expectation is taken with respect to s and b.

<u>Proof</u>: First note that from lemma 1, for (t,x) to be individually rational it must be that $U(\bar{s}) \geq 0$ and $V(\underline{b}) \geq 0$. For the seller, we have

$$\int_{\underline{s}}^{\bar{s}} U(s)f(s)ds = U(\bar{s}) + \int_{\underline{s}}^{\bar{s}}\int_{s}^{\bar{s}} P(u)du\ f(s)ds \qquad \text{(US)}$$

$$= U(\bar{s}) + \int_{\underline{s}}^{\bar{s}} F(s)P(s)ds$$

$$= U(\bar{s}) + \int_{\underline{b}}^{\bar{b}}\int_{\underline{s}}^{\bar{s}} F(s)e^{-pt(s,b)}g(b)ds\ db$$

where the first equality follows from lemma 1 and integration by parts, and the second equality results from changing the order of integration. Similarly for the buyer, we have

$$\int_{\underline{b}}^{\bar{b}} V(b)g(b)db = V(b) + \int_{\underline{b}}^{\bar{b}}\int_{\underline{s}}^{\bar{s}} [1 - G(b)]e^{-\sigma t(s,b)}f(s)ds\ db \quad . \qquad \text{(UB)}$$

Rearranging terms in (US) and (UB) and substituting the definitions for $U(s)$ and $V(b)$, results in the desired expression (IR) for $U(\bar{s}) + V(\underline{b})$. Q.E.D.

<u>Lemma 4</u>: If the function $t(\cdot,\cdot)$ is such that P is decreasing, Q is increasing, and (IR) is satisfied, then there exists a function $x(\cdot,\cdot)$, such that $\langle t,x \rangle$ is incentive compatible and individually rational.

<u>Proof</u>: The proof is by construction. Let

$$x(s,b) = \int_{\underline{b}}^{b} u\ dQ(u) + \int_{\underline{s}}^{s} u\ dP(u) + c$$

where c is a constant chosen so that $V(\underline{b}) = 0$. To compute c, notice that

$$V(\underline{b}) = \underline{b}Q(\underline{b}) - \int_{\underline{s}}^{\bar{s}} x(s,\underline{b})f(s)ds$$

$$= \underline{b}Q(\underline{b}) - c - \int_{\underline{s}}^{\bar{s}} \int_{\underline{s}}^{s} u \, dP(u)f(s)ds$$

$$= \underline{b}Q(\underline{b}) - c + \int_{\underline{s}}^{\bar{s}} s[1 - F(s)]dP(s) = 0 \quad .$$

Thus,

$$c = \underline{b}Q(\underline{b}) + \int_{\underline{s}}^{\bar{s}} s[1 - F(s)]dP(s) \quad .$$

Incentive compatibility for the seller is verified by showing that the seller s is better off reporting s than $s' \neq s$: for all s, $s' \in [\underline{s},\bar{s}]$,

$$s[P(s') - P(s)] + S(s) - S(s') = s\int_{s}^{s'} dP(u) - \int_{s}^{s'} u \, dP(u)$$

$$= \int_{s}^{s'} [s - u]dP(u) \geq 0$$

since P is decreasing. An identical argument holds for the buyer.

Since $V(\underline{b}) = 0$ and $\langle t,x \rangle$ is incentive compatible and satisfies

(IR), it follows from lemma 3 that $U(\bar{s}) > 0$. Thus, the bargaining mechanism <t,x> is incentive compatible and individually rational. Q.E.D.

Sequential Rationality

To understand how learning takes place in a sequential bargaining mechanism, it is best to interpret the direct revelation game as follows. At time zero (after the players know their private information), the players agree to adopt a particular sequential bargaining mechanism <t,x> that is interim efficient. (Note that any interim efficient mechanism can be chosen as a Nash equilibrium in an appropriately defined "choice of mechanism" game.) The players then report their private information in sealed envelopes to a mediator, who will then implement the mechanism <t,x>. (Actually, a third party is not necessary, since the role of the mediator can be carried out by a computer that is programmed by the bargainers to execute the mechanism.) After opening the envelopes, the mediator does not announce the outcome immediately by saying something like "Trade shall occur two months from now at the price of one thousand dollars," but instead waits until two months have past and <u>then</u> announces "Trade shall occur now at the price of one thousand dollars." It is necessary that the mediator wait until the time of trade in order for the mechanism to be sequentially rational, since otherwise the bargainers would have an incentive to ignore the mediator's announcement and trade immediately.

As time passes, the players are able to refine their inferences about the other player's private information based on the information that the mediator has not yet made an announcement. Initially, it is

common knowledge that the players' valuations are distributed according to the probability distributions F and G, but after τ units of time have elapsed the common knowledge beliefs become the distributons F and G conditioned on the fact that an announcement has not yet been made:

$$F_\tau(s) = F(s|t(s,b) > \tau) \quad ; \quad G_\tau(b) = G(b|t(s,b) > \tau) \quad .$$

Thus, at any time $\tau > 0$, the mechanism $<t,x>$ induces an outcome function $t(s,b) = t(s,b|F_\tau,G_\tau)$ for all s and b. A mechanism $<t,x>$ is sequentially rational if at every time $\tau > 0$ the induced outcome function $t(s,b|F_\tau,G_\tau)$ is interim efficient; that is, there does not exist a mechanism $<t',x'>$ preferable to $<t,x>$ at some time $\tau > 0$ for all remaining traders and strictly preferred by at least one trader.

The following lemma relates the definition of sequentially rational to common knowledge dominance.

Lemma 5: A sequential bargaining mechanism $<t,x>$ is sequentially rational if and only if it is never common knowledge that the mechanism $t(\cdot,\cdot|F_\tau,G_\tau)$ it induces over time is dominated by an alternative mechanism.

Proof: From Theorem 1 of Holmstrom and Myerson [1983], we know that a mechanism is interim efficient if and only if it is not common knowledge dominated by any other incentive-compatible and individually-rational mechanism. Q.E.D.

A necessary condition for a mechanism to be sequentially rational is for the bargainers to continue negotiations so long as each expects positive gains from continuing. For the model here, since there are no transaction costs (only delay costs), this means that negotiations cannot end if there exists a pair of players that have not yet come to an agreement, but for which agreement is beneficial at some point in the future. Formally, for the bargaining mechanism $<t,x>$ to be sequentially rational it must be that for all potential players, a failure to reach agreement implies that there is some point beyond which agreement is never beneficial: for all s and b,

$$t(s,b) = \infty \rightarrow \exists\ \hat{\tau} > 0 \text{ such that } \forall\ \tau > \hat{\tau}\ ,\quad s > be^{(\rho-\sigma)\tau}\ .$$

The condition $s > be^{(\rho-\sigma)\tau}$ is simply a statement that trade is not beneficial at time τ, since

$$x - se^{-\rho\tau} + be^{-\sigma\tau} - x > 0 \leftrightarrow se^{-\rho\tau} > be^{-\sigma\tau} \leftrightarrow s > be^{(\rho-\sigma)\tau}\ .$$

Notice that the strength of ths requirement depends on the relative magnitudes of the players' discount rates. When $\rho > \sigma$, then $e^{(\rho-\sigma)\tau} \rightarrow \infty$ as $\tau \rightarrow \infty$, so for all potential pairs of players it is never the case that there exists a time at which trade is never beneficial in the future. Thus, when $\rho > \sigma$, the mechanism $<t,x>$ is sequentially rational only if trade always occurs: $t(s,b) < \infty\ \forall\ s,b$. Likewise, when $\rho > \sigma$, then $e^{(\rho-\sigma)\tau} \rightarrow 0$ as $\tau \rightarrow \infty$, so for every pair of players there is always a point at which trade becomes undesirable

for all times in the future. Finally, if $\rho = \sigma$ then the necessary condition for sequential rationality becomes $t(s,b) = \infty \rightarrow s > b$: trade must occur whenever the gains from trade are initially positive.

In order to state this necessary condition in a lemma, it will be useful to define the set B to be the set of potential traders for which trade is always beneficial at some time in the future:

$$B = \{(s,b) \mid \rho > \sigma \text{ or } (\rho = \sigma \text{ and } s < b)\} .$$

Lemma 6: Any mechanism <t,x> that excludes trade over a nonempty subset of B violates sequential rationality.

Proof: Let $N \subseteq B$ be the set for which trade never occurs. Then at some point τ the induced mechanism has $t(s,b|F_\tau,G_\tau) = \infty$ for all remaining traders, which includes N. But this mechanism is not interim efficient, since it is dominated by a mechanism that results in a positive probability of trade for some traders in N (a partially pooling equilibrium with this property will always exist). Q.E.D.

I claim that sequential rationality is a necessary condition for rationality in games with incomplete information in which commitment is not possible. If a mechanism is not sequentially rational, then at some point in time it is common knowledge that all potential agents would prefer an alternative mechanism and hence this alternative mechanism will be adopted by the agents at that point in time. Thus, it would be inconsistent for the players to believe that the original mechanism would be carried out faithfully.

Necessary and Sufficient Conditions for Perfection

Lemmas 1-5 are summarized in the following theorem, which gives necessary and sufficient conditions for the sequential bargaining mechansim $\langle t,x\rangle$ to be perfect.

Theorem 1: A sequential bargaining mechanism $\langle t,x\rangle$ is incentive compatible if and only if the functions

$$S(s) = \int_{\underline{b}}^{\bar{b}} x(s,b)g(b)db \quad ; \quad P(s) = \int_{\underline{b}}^{\bar{b}} e^{-\rho t(s,b)} g(b)db$$

$$B(b) = \int_{\underline{s}}^{\bar{s}} x(s,b)f(s)ds \qquad Q(b) = \int_{\underline{s}}^{\bar{s}} e^{-\sigma t(s,b)} f(s)ds$$

are such that P is decreasing, Q is increasing, and

$$S(s) - S(\bar{s}) = \int_{s}^{\bar{s}} - u \; dP(u) \quad ; \quad B(b) - B(\underline{b}) = \int_{\underline{b}}^{b} u \; dQ(u) \quad . \qquad \text{(IC)}$$

Furthermore, for t such that P is decreasing and Q is increasing, there exists an x such that $\langle t,x\rangle$ is incentive compatible and individually rational if and only if

$$U(\bar{s}) + V(\underline{b}) = \varepsilon\{[b - \frac{1 - G(b)}{g(b)}]e^{-\sigma t(s,b)} - [s + \frac{F(s)}{f(s)}]e^{-\rho t(s,b)}\} \geq 0 \quad . \text{(IR)}$$

Finally, the mechanism $\langle t,x\rangle$ is sequentially rational if and only if it is never common knowledge that the mechanism it induces over time is dominated by an alternative mechanism.

4. Efficiency

The set of perfect bargaining mechanisms is typically quite large, which means there are many extensive-form games with equilibria satisfying incentive compatibility, individual rationality, and sequential rationality. To narrow down this set, it is natural to assumed additional efficiency properties. Three notions of efficiency, described at length by Holmstrom and Myerson [1983], are ex post, interim, and ex ante efficiency. The difference among these concepts centers on what information is available at the time of evaluation: ex ante efficiency assumes that comparisons are made before the players know their private information; interim efficiency assumes that the players know only their private information; and ex post efficiency assumes that all information is known.

Ex Post Efficiency

Ideally one would like to find perfect bargaining mechanisms that are ex post efficient. The mechanism $\langle t,x\rangle$ is ex post efficient if there does not exist an alternative mechanism that can make both players better off in terms of their ex post utilities (after all information is revealed).[3/] Equivalently, for a mechanism to be ex post efficient, it must maximize a weighted sum $\alpha_1(s,b)u(s) + \alpha_2(s,b)v(b)$ of the players' ex post utilities for all s and b, where $\alpha_1(\cdot,\cdot), \alpha_2(\cdot,\cdot) \geq 0$ and the ex post utilities of seller s and buyer b are

$$u(s,b) = x(s,b) - se^{-\rho t(s,b)} \quad ; \quad v(s,b) = be^{-\sigma t(s,b)} - x(s,b) \quad .$$

Since the payoff functions are additively separable in money and goods, and thus utility is transferable between players, we can assume equal weights $(\alpha_1(s,b) = \alpha_2(s,b) = 1 \ \forall \ s,b)$ without loss of generality. In order to simplify notation, define $p(s,b) = e^{-t(s,b)}$, so that $p(s,b)^\rho$ is the discounted probability of agreement for seller s given that the buyer has valuation b, and $p(s,b)^\sigma$ is the discounted probability of agreement for buyer b given the seller has cost s. With this change, a sequential bargaining mechanism becomes the pair of functions $\langle p,x \rangle$ where $p\colon [\underline{s},\bar{s}] \times [\underline{b},\bar{b}] \to [0,1]$. The bargaining mechanism $\langle p,x \rangle$, then, is ex post efficient if for all $s \in [\underline{s},\bar{s}]$ and $b \in [\underline{b},\bar{b}]$, the function $p(s,b)$ is chosen to solve the program

$$\max_{p\in[0,1]} \ \pi(p) = bp^\sigma - sp^\rho \quad .$$

The first-order condition is

$$\frac{d\pi}{dp} = \sigma b p^{\sigma-1} - \rho s p^{\rho-1} = 0$$

or

$$p = \left(\frac{\sigma b}{\rho s}\right)^{\frac{1}{\rho-\sigma}} \quad .$$

Checking the boundary conditions and assuming $\underline{s},\underline{b} > 0$, yields

$$p^*(s,b) = \begin{cases} 1 & \text{if } s < b \;,\; \rho s \leq \sigma b \\ \left(\frac{\sigma b}{\rho s}\right)^{\frac{1}{\rho-\sigma}} & \text{if } \rho > \sigma \;,\; \rho s > \sigma b \\ 0 & \text{if } \rho \leq \sigma \;,\; s \geq b^{\cdot} \end{cases}$$

The following theorem demonstrates that it is impossible to find ex post efficient mechanisms, if the bargainers are uncertain about whether or not trade should occur immediately. This result is shown in an example in Cramton [1983].

Theorem 2: There exists an incentive-compatible individually-rational bargaining mechanism that is ex post efficient if it is common knowledge that trade should occur immediately. However, an ex post efficient mechanism does not exist if the buyer's delay cost is at least as great as the seller's and it is not common knowledge that gains from trade exist.

Proof: Suppose that it is common knowledge that trade should occur immediately. Then three cases are possible: (1) $\rho \leq \sigma$ and $\bar{s} \leq \underline{b}$, (2) $\rho > \sigma$ and $\rho\bar{s} \leq \sigma\underline{b}$, and (3} $\rho = \infty$ and $\sigma < \infty$. I need to show that $p^*(s,b) = 1$ for all s,b satisfies (IR). For cases 1 and 2,

$$
\begin{aligned}
U(\bar{s}) + V(\underline{b}) &= \varepsilon\{[b - \frac{1 - G(b)}{g(b)}] - [s + \frac{F(s)}{f(s)}]\} \\
&= \varepsilon\{b - \frac{1 - G(b)}{g(b)}\} - \varepsilon\{s + \frac{F(s)}{f(s)}\} \\
&= \int_{\underline{b}}^{\bar{b}} [bg(b) - 1 + G(b)]db - \int_{\underline{s}}^{\bar{s}} [sf(s) + F(s)]ds \\
&= bG(b)\Big|_{\underline{b}}^{\bar{b}} - \bar{b} + \underline{b} - sF(s)\Big|_{\underline{s}}^{\bar{s}} \\
&= \underline{b} - \bar{s} > 0
\end{aligned}
$$

where the integration is done by parts. In case 3,

$$
U(\bar{s}) + V(\underline{b}) = \varepsilon\{b - \frac{1 - G(b)}{g(b)}\} = \underline{b} > 0 \quad .
$$

Then by lemma 4, there exists an x such that $\langle p,x\rangle$ is incentive compatible and individually rational.

Now assume that it is not common knowledge that gains from trade exist and the buyer's delay cost is at least as great as the seller's ($\rho \leq \sigma$). Notice that when $\rho \leq \sigma$ we get that $\langle p,x\rangle$ is ex post efficient if trade occurs without delay whenever there are positive gains from trade:

$$
p^*(s,b) = \begin{cases} 1 & \text{if} \quad s < b \\ 0 & \text{if} \quad s \geq b \end{cases} .
$$

Substituting this function for p into (IR) yields

$$U(\bar{s}) + V(\underline{b})$$

$$= \int_{\underline{b}}^{\bar{b}} \int_{\underline{s}}^{\min\{b,\bar{s}\}} [bg(b) + G(b) - 1]f(s)ds\ db - \int_{\underline{b}}^{\bar{b}} \int_{\underline{s}}^{\min\{b,\bar{s}\}} [sf(s) + F(s)]ds\ g(b)db$$

$$= \int_{\underline{b}}^{\bar{b}} [bg(b) + G(b) - 1]F(b)db - \int_{\underline{b}}^{\bar{b}} \min\{bF(b),\bar{s}\}g(b)db$$

$$= - \int_{\underline{b}}^{\bar{b}} [1 - G(b)]F(b)db + \int_{\underline{s}}^{\bar{b}} (b - \bar{s})g(b)db$$

$$= - \int_{\underline{b}}^{\bar{b}} [1 - G(b)]F(b)db + \int_{\bar{s}}^{\bar{b}} [1 - G(b)]db$$

$$= - \int_{\underline{b}}^{\bar{s}} [1 - G(u)]F(u)du \quad .$$

Thus, any incentive-compatible mechanism that is ex post efficient must have

$$U(\bar{s}) + V(\underline{b}) = - \int_{\underline{b}}^{\bar{s}} [1 - G(u)]F(u)du < 0$$

and so it cannot be individually rational. Q.E.D.

When the seller's delay cost is greater than the buyer's and it is not common knowledge that trade should occur immediately, a general proof that ex post efficiency is not achievable cannot be given due to the complicated expression for $p^*(s,b)$ in this case. However,

analysis of examples (see section 5) suggests that ex post efficiency is typically unobtainable.

Ex Ante Efficiency

The strongest concept of efficiency, other than ex post efficiency (which is generally unobtainable), that can be applied to games of incomplete information is ex ante efficiency. A player's ex ante utility is his expected utility before he knows his type. Thus, given the sequential bargaining mechanism $\langle p,x \rangle$, the seller's and buyer's ex ante utilities are

$$\bar{U} = \int_{\underline{s}}^{\bar{s}} U(s)f(s)ds = \int_{\underline{s}}^{\bar{s}} \int_{\underline{b}}^{\bar{b}} [x(s,b) - sp(s,b)^{\rho}]g(b)db \; f(s)ds$$

$$\bar{V} = \int_{\underline{b}}^{\bar{b}} V(b)g(b)db = \int_{\underline{b}}^{\bar{b}} \int_{\underline{s}}^{\bar{s}} [bp(s,b)^{\sigma} - x(s,b)]f(s)ds \; g(b)db \quad .$$

The mechanism $\langle p,x \rangle$ is ex ante efficient if there does not exist an alternative mechanism that can make both players better off in terms of their ex ante utilities. Thus, for a mechanism to be ex ante efficient, it must maximize a weighted sum $\alpha_1 \; + \alpha_2$ of the players' ex ante utilities, where $\alpha_1, \alpha_2 > 0$. For tractability and reasons of equity, I will assume equal weights $(\alpha_1 = \alpha_2 = 1)$.[4/] The use of unequal weights would not significantly change the results, but would greatly complicate the analysis.

If the bargainers were to choose a bargaining mechanism before

they knew their types, it would seem reasonable that they would agree to a scheme that was ex ante efficient. It is generally the case, however, that the players know their private information before they begin negotiatons, and therefore would be unable to agree on an ex ante efficient mechanism, since the players are concerned with their <u>interim</u> utilities $U(s)$ and $V(b)$ rather than their ex ante utilities $\bar{U}$ and $\bar{V}$. Nevertheless, it may be that the sequential bargaining mechanism is chosen by an uninformed social planner or arbitrator in which case the selection of an ex ante efficient mechanism would be justified. Alternatively, one might suppose that the choice of a bargaining mechanism is based on established norms of behavior and that these norms have evolved over time in such a way as to produce ex ante efficient mechanisms. In situations where the choice of a bargaining mechanism does not occur before the players know their types or is not handled by an uninformed third party, such as an arbitrator, ex ante efficiency is too strong a requirement. The weaker requirement of <u>interim efficiency</u>, which requires that there does not exist a dominating mechanism in terms of the player's interim utilities $U(s)$ and $V(b)$, is more appropriate.

The sum of the players' ex ante utilities for the bargaining mechanism $\langle p,x\rangle$ is given by

$$\bar{U} + \bar{V} = \int_{\underline{b}}^{\bar{b}} \int_{\underline{s}}^{\bar{s}} [bp(s,b)^{\sigma} - sp(s,b)^{\rho}] f(s)g(b)ds\, db \quad .$$

A bargaining mechanism, then, is ex ante efficient if it maximizes this sum subject to incentive compatibility and individual rationality:

$$\max_{p(\cdot,\cdot)} \quad \varepsilon\{bp(s,b)^{\sigma} - sp(s,b)^{\rho}\} \quad \text{such that}$$

$$\varepsilon\{[b - \frac{1 - G(b)}{g(b)}]p(s,b)^{\sigma} - [s + \frac{F(s)}{f(s)}]p(s,b)^{\rho}\} \geq 0 \quad , \tag{P}$$

where p is chosen so that P is decreasing and Q is increasing. Multiplying the constraint by $\lambda \geq 0$ and adding it to the objective function, yields the Lagrangian

$$L(p,\lambda) = \varepsilon\{((1 + \lambda)b - \lambda \frac{1 - G(b)}{g(b)})p(s,b)^{\sigma} - ((1 + \lambda)s + \lambda\frac{F(s)}{f(s)})p(s,b)^{\rho}\}$$

$$= (1 + \lambda)\varepsilon\{(b - \frac{\lambda}{1 + \lambda} \frac{1 - G(b)}{g(b)})p(s,b)^{\sigma} - (s + \frac{\lambda}{1 + \lambda} \frac{F(s)}{f(s)})p(s,b)^{\rho}\} \quad .$$

For any $\alpha \geq 0$, define the functions

$$c(s,\alpha) = s + \alpha \frac{F(s)}{f(s)} \qquad d(b,\alpha) = b - \alpha \frac{1 - G(b)}{g(b)} \quad .$$

Then the Lagrangian (ignoring the constant $(1 + \lambda)$) becomes

$$L(p,\lambda) = \varepsilon\{d(b,\alpha)p(s,b)^{\sigma} - c(s,\alpha)p(s,b)^{\rho}\} \quad ,$$

which is easily maximized by pointwise optimization. The first-order condition is

$$\frac{d}{dp} = \sigma dp^{\sigma-1} - \rho cp^{\rho-1}$$

or

$$p = (\frac{\sigma d}{\rho c})^{\frac{1}{\rho-\sigma}} \quad .$$

Establishing the boundary conditions and noticing that $c(\cdot,\cdot) > 0$, yields the optimal solution

$$p_\alpha(s,b) = \begin{cases} 1 & \text{if } c(s,\alpha) < d(b,\alpha) \quad , \quad \rho c(s,\alpha) < \sigma d(b,\alpha) \\ \left(\frac{\sigma d(b,\alpha)}{\rho c(s,\alpha)}\right)^{\frac{1}{\rho-\sigma}} & \text{if } \rho > \sigma \quad , \quad \rho c(s,\alpha) > \sigma d(b,\alpha) > 0 \\ 0 & \text{if } [p < \sigma, c(c,\alpha) < d(b,\alpha)] \text{ or } d(b,\alpha) < 0 \end{cases} .$$

The following theorem determines how to find an ex ante efficient mechanism for any sequential bargaining game.

Theorem 3: If there exists an incentive-compatible mechanism $<p,x>$ such that $p = p_\alpha$ for some α in $[0,1]$ and $U(\bar{s}) = V(\underline{b}) = 0$, then this mechanism is ex ante efficient. Moreover, if $c(\cdot,1)$ and $d(\cdot,1)$ are increasing functions on $[\underline{s},\bar{s}]$ and $[\underline{b},\bar{b}]$ respectively, and ex post efficiency is unobtainable, then such a mechanism must exist.

Proof: The first sentence of this theorem follows from the fact that the Lagrangian $L(p,\lambda)$ is maximized by the function p_α with $\alpha = \lambda/1 + \lambda$. Hence, p_α yields an ex ante efficient mechanism provided the individual-rationality constraint is binding.

To prove the existence part of the theorem, suppose that $c(\cdot,1)$ and $d(\cdot,1)$ are increasing, and that the players are uncertain about whether or not trade should occur immediately. Then for every α $[0,1]$, $c(\cdot,1)$ and $d(\cdot,1)$ are increasing, which implies that $p_\alpha(s,b)$ is increasing in s and decreasing in b. Thus, P is

decreasing and Q is increasing as required by incentive and compatibility.

It remains to be shown that there is a unique $\alpha \in [0,1]$, for which the individual-rationality constraint is binding. Define

$$R(\alpha) = \mathcal{E}\{d(b,1)[p_\alpha(s,b)]^\sigma - c(s,1)[p_\alpha(s,b)]^\rho\}$$

so that $R(\alpha)$ is the value of the integral in the individual-rationality constraint as a function of α. First, notice that $R(1) \geq 0$, since the term in the expectation is nonnegative for all s and b. Furthermore, $R(0) < 0$, since there does not exist an ex post efficient mechanism. Therefore, if $R(\alpha)$ is continuous and strictly increasing in α, then there is a unique $\alpha \in [0,1]$ for which $R(\alpha) = 0$.

The continuity and monotonicity of $R(\cdot)$ are most easily verified by considering two cases.

<u>Case 1</u>: $(\rho \leq \sigma)$. When $\rho \leq \sigma$, then

$$p_\alpha(s,b) = \begin{cases} 1 & \text{if} \quad c(s,\alpha) < d(b,\alpha) \\ 0 & \text{if} \quad c(s,\alpha) \geq d(b,\alpha) \end{cases} .$$

Thus, $p_\alpha(s,b)$ is decreasing in α, since

$$d(b,\alpha) - c(s,\alpha) = (b - s) - \alpha(\frac{1 - G(b)}{g(b)} + \frac{F(s)}{f(s)})$$

is decreasing in α. Thus, for $\alpha < \beta$, $R(\beta)$ differs from $R(\alpha)$ only because $0 = p^\beta(s,b) < p_\alpha(s,b) = 1$ for some (s,b) where

$d(b,\beta) < c(s,\beta)$ and so $d(b,1) < c(s,1)$. Therefore, $R(\cdot)$ is stictly increasing.

To prove $R(\cdot)$ is continuous, observe that, if $c(s,1)$ and $d(b,1)$ are increasing in s and b, then $c(\cdot,\alpha)$ and $d(\cdot,\alpha)$ are strictly increasing for any $\alpha < 1$. So given b and α, the equation $c(s,\alpha) = d(b,\alpha)$ has at most one solution in s, and this solution varies continuously in b and α. Hence, we may write

$$R(\alpha) = \int_{\underline{b}}^{\bar{b}} \int_{\underline{s}}^{r(b,\alpha)} \big(d(b,1) - c(s,1)\big) f(s) g(b)\, ds\, db$$

where $r(b,\alpha)$ is continuous in b and α. Thus, $R(\alpha)$ is continuous in α.

Case 2: $(\rho > \sigma)$. When $\rho > \sigma$, then

$$p_\alpha(s,b) = \begin{cases} 1 & \text{if } \rho c(s,\alpha) \leq \sigma d(b,\alpha) \\ \left(\dfrac{\sigma d(b,\alpha)}{\rho c(s,\alpha)}\right)^{\frac{1}{\rho-\sigma}} & \text{if } \rho c(s,\alpha) > \sigma d(b,\alpha) > 0 \\ 0 & \text{if } d(b,\alpha) \leq 0 \ . \end{cases}$$

Since

$$\sigma d(b,\alpha) - \rho c(s,\alpha) = \sigma b - \rho s - \alpha\left(\sigma \frac{1 - G(b)}{g(b)} + \frac{F(s)}{f(s)}\right) ,$$

$$\frac{\sigma d(b,\alpha)}{\rho c(s,\alpha)} = \frac{\sigma}{\rho} \left(\frac{b - \alpha \frac{1 - G(b)}{g(b)}}{s + \alpha \frac{F(s)}{f(s)}}\right) ,$$

and $d(b,\alpha)$ are decreasing in α, $p_\alpha(s,b)$ is decreasing in α. Thus, for $\alpha < \beta$, $R(\beta)$ differs from $R(\alpha)$ only because $p^\beta(s,b) < p_\alpha(s,b)$

where $\sigma d(b,\alpha) < \rho c(s,\alpha)$. Therefore, $R(\cdot)$ is strictly increasing.

Since $c(\cdot,\alpha)$ and $d(\cdot,\alpha)$ are strictly increasing for any $\alpha < 1$, the equation $d(b,\alpha) = 0$ has at most one solution in b and the equation $\rho c(s,\alpha) = \sigma d(b,\alpha)$ has at most one solution in s, and the solutions vary continuously in b and α. Hence, we may write

$$R(\alpha) = \int_{q(\alpha)}^{\bar{b}} \Big[\int_{\underline{s}}^{r(b,\alpha)} [d(b,1) - c(b,1)]f(s)ds$$

$$+ \int_{r(b,\alpha)}^{\bar{s}} \big(d(b,1)[p_\alpha(s,b)]^\sigma - c(s,1)[p_\alpha(s,b)]^\rho\big)f(s)ds\Big]g(b)db$$

where $q(\alpha)$ and $r(b,\alpha)$ are continuous in b and α. Therefore, $R(\alpha)$ is continuous in α.

Since $R(\cdot)$ is continuous and strictly increasing with $R(0) < (0)$ and $R(1) > 0$, there must be a unique α $[0,1]$ such that $R(\alpha) = 0$ and $p_\alpha(s,b)$ is ex ante efficient. Q.E.D.

It is worthwhile to point out that the requirement in the existence part of theorem 3 that $c(\cdot,1)$ and $d(\cdot,1)$ be increasing functions is satisfied by a large range of distribution functions. A sufficient condition for $c(\cdot,1)$ and $d(\cdot,1)$ to be increasing is for the ratio of the distribution and the density to be increasing. This is a local characterization of the monotone likelihood ratio property and is satisfied by many distributions, such as the uniform, exponential, normal, chi-square, and Poisson distributions.

I now prove that the ex ante efficient mechanism typically violates sequential rationality, and hence show that bargainers who are unable to make binding commitments are worse off (in an ex ante sense) than those bargainers able to commit to particular strategies.

Corollary 1: If ex post efficiency is unobtainable, $c(\cdot,1)$ and $d(\cdot,1)$ are increasing functions, and $d(\underline{b},1) < 0$ if $\rho > \sigma$, then the ex ante efficient mechanism violates sequential rationality.

Proof: By theorem 3, the ex ante efficient mechanism exists and is given by p_α for some $\alpha > 0$. Consider the set of traders that never trade under p_α, but for which trade is always beneficial at some point in the future:

$$N = \{(s,b) | p_\alpha(s,b) = 0 \text{ and } [\rho = \sigma \text{ and } s < b)]\} .$$

By our hypothesis, this set is nonempty. Thus, from lemma 6, the mechanism p_α violates sequential rationality. Q.E.D.

5. The Case of Uniform Symmetric Exchange: An Example

In order to illustrate the theory presented in the earlier sections, it will be useful to look at an example. In particular, consider the case of uniform symmetric exchange in which both the seller's cost and the buyer's valuation are uniformly distributed on $[0,1]$. Then $c(s,\alpha) = (1 + \alpha)s$ and $d(b,\alpha) = (1 + \alpha)b - \alpha$, which are strictly increasing when $\alpha = 1$, so by theorem 3 we know that, for some $\alpha \in [0,1]$, the mechanism $p = p_\alpha$ is ex ante efficient. The desired

α is found by setting $R(\alpha)$ to zero, so that $U(\bar{s}) = V(\underline{b}) = 0$. Again, it will be useful to consider two cases depending on whether $\rho < \sigma$ or $\rho > \sigma$.

<u>Case 1</u>: $(\rho < \sigma)$. When $\rho < \sigma$, then

$$p_\alpha(s,b) = \begin{cases} 1 & \text{if} \quad s < b - \frac{\alpha}{1+\alpha} \\ 0 & \text{if} \quad s > b - \frac{\alpha}{1+\alpha} \end{cases} .$$

Define $\mu = \alpha/(1 + \alpha)$ Then we wish to find $\mu \in [0,1/2]$ such that

$$R(\alpha) = \int_\mu^1 \int_0^{b-\mu} [2(b - s) - 1] ds\, db = 0 \quad .$$

Performing the integration yields

$$(\mu - \tfrac{1}{4})(\mu + 1)^2 = 0$$

which has a root in $[0,1/2]$ at $\mu = 1/4$. Thus, $\alpha = 1/3$ and

$$p(s,b) = \begin{cases} 1 & \text{if} \quad s < b - \frac{1}{4} \\ 0 & \text{if} \quad s > b - \frac{1}{4} \end{cases} .$$

When $\rho < \sigma$, ex ante efficiency is obtained by a mechanism that transfers the object without delay if and only if the buyer's valuation exceeds the seller's by at least 1/4. Perhaps somewhat surprisingly, the ex ante efficient mechanism in this case does not depend on ρ or σ. Since the value of the object is declining more rapidly for the buyer than the seller, it is always better to transfer the item immediately if at all. Hence, even though the players can reveal information by

delaying agreement, in the ex ante efficient mechanism they choose to trade immediately or not at all, so that a static mechanism ex ante dominates any sequential bargaining mechanism. This static mechanism, however, is not sequentially rational, which illustrates Corollary 1.

An extensive-form game that implements the ex ante efficient mechanism, when $\rho < \sigma$, has been studied by Chatterjee and Samuelson [1983]. They consider the simultaneous-offer game, in which the players simultaneously announce prices and the object is traded if the buyer's bid exceeds the seller's offer. For this example, the seller's optimal strategy is to offer the price $(2/3)s + (1/4)$ and the buyer's best response is to bid $(2/3)b + (1/12)$, which implies that trade occurs provided $(2/3)s + (1/4) < (2/3)b + (1/12)$ or $s < b - (1/4)$ as in the ex ante efficient mechanism. For this equilibrium, the price at which the object is sold is

$$x(s,b) = \begin{cases} \frac{1}{3}(b + s) + 1/6 & \text{if} \quad s < b - \frac{1}{4} \\ 0 & \text{if} \quad s > b - \frac{1}{4} \end{cases} .$$

The sume of the players' ex ante utilities is

$$\bar{U} + \bar{V} = \int_{1/4}^{1} \int_{0}^{b-(1/4)} (b - s)ds\, db = \frac{9}{64} ,$$

whereas the total utility from the ex post efficient mechanism is

$$\int_0^1 \int_0^b (b - s)ds\, db = \frac{1}{6} .$$

Thus, 15.6% of the gains from trade are lost, when $\rho < \sigma$, due to delays in agreement.

Case 2: $(\rho > \sigma)$. When $\rho > \sigma$, then

$$p_\alpha(s,b) = \begin{cases} 1 & \text{if } s < \frac{\sigma}{\rho}(b - \frac{\alpha}{1+\alpha}) \\ \left(\frac{\sigma(b - \frac{\alpha}{1+\alpha})}{\rho s}\right)^{1/(\rho-\sigma)} & \text{if } s > \frac{\sigma}{\rho}(b - \frac{\alpha}{1+\alpha}) \\ 0 & \text{if } b < \frac{\alpha}{1+\alpha} \end{cases} .$$

Making the substitution $\mu = \alpha/(1 + \alpha)$, we wish to find $\mu \quad [0,1/2]$ such that

$$\int_\mu^1 \Big[\int_0^{\frac{\sigma}{\rho}(b-\mu)} (2b - 1 - 2s)ds + \int_{\frac{\sigma}{\rho}(b-\mu)}^1 \Big[(2b - 1) \frac{\sigma(b - \mu)}{\rho s}^{\frac{\sigma}{\rho-\sigma}}$$

$$- 2s \frac{\sigma(b - \mu)}{\rho s}^{\frac{\rho}{\rho-\sigma}}\Big] ds \Big]db = 0 .$$

Let $\delta = \sigma/\rho$ and $\gamma = \rho/(\rho - \sigma)$. After this substitution, we have

$$\int_\mu^1 \Big[\int_0^{\delta(b-\mu)} (2b - 1 - 2s)ds + \int_{\delta(b-\mu)}^1 \Big[(2b - 1)[\delta(b - \mu)]^{\gamma}$$

$$- 2[\delta(b - \mu)]^{1+\gamma}\Big]s^{-\gamma}ds\Big]db = 0 .$$

Performing the inner integration (assuming $\gamma \neq 1$) yields

$$\int_{\mu}^{1}[\delta(b - \mu)[(2 - \delta)b + \delta\mu - 1]$$

$$+ \frac{1}{1 - \gamma}[2(1 - \delta)b + 2\delta\mu - 1][\delta(b - \mu)]^{\gamma}\left(1 - [\delta(b - \mu)]^{1-\gamma}\right)]db$$

$$= \int_{\mu}^{1}[\delta\left((2 - \delta)b^2 - [2\mu(1 - \delta) + 1]b + \mu(1 - \delta\mu)\right)$$

$$+ \frac{1}{1 - \gamma}[2(1 - \delta)b + 2\delta\mu - 1]\left([\delta(b - \mu)]^{\gamma} - \delta(b - \mu)\right)]db$$

$$= \int_{\mu}^{1} \frac{\delta}{1 - \gamma} \left[\left(\delta - \gamma(2 - \delta)\right)b^2 - \left(\gamma[1 + 2\mu(1 - \delta)] - 2\delta\mu\right)b\right.$$

$$\left. + \delta\mu^2 - \gamma\mu(1 - \delta\mu) + [2(1 - \delta)b + 2\delta\mu - 1]\delta^{\gamma-1}(b - \mu)^{\gamma}\right]db = 0 \quad .$$

Since

$$\int_{\mu}^{1}(b - \mu)^{\gamma}db = \frac{(1 - \mu)^{1+\gamma}}{1 + \gamma} \quad \text{and} \quad \int_{\mu}^{1}b(b - \mu)^{\gamma}db = \frac{(1 - \mu)^{1+\gamma}}{1 + \gamma}\left(1 - \frac{1 - \mu}{2 + \gamma}\right) \quad ,$$

after integration we have

$$\frac{\delta}{1 - \gamma}\left[\tfrac{1}{3}(1 - \mu^3)(\delta - \gamma(2 - \delta)) + \tfrac{1}{2}(1 - \mu^2)(\gamma[1 + 2\mu(1 - \delta)] - 2\delta\mu]\right.$$

$$+ (1 - \mu)(\delta\mu^2 - \gamma\mu(1 - \delta\mu)) + \frac{\delta^{\gamma-1}(1 - \mu)^{1+\gamma}}{1 + \gamma}\left(2\delta\mu - 1\right.$$

$$\left.\left. + 2(1 - \delta)\left[1 - \frac{1 - \mu}{2 + \gamma}\right)\right]\right] = 0 \quad .$$

Dividing by $\delta(1 - \mu)/(1 - \gamma)$, yields

Figure 1

μ as a function of the ratio of the player's discount rates

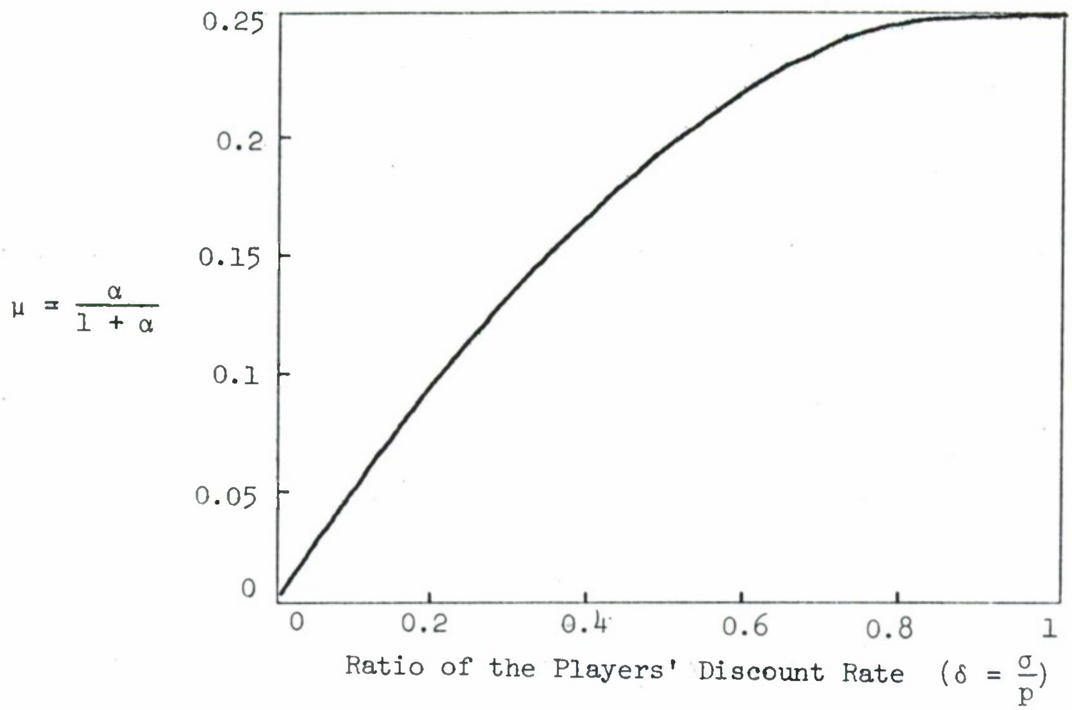
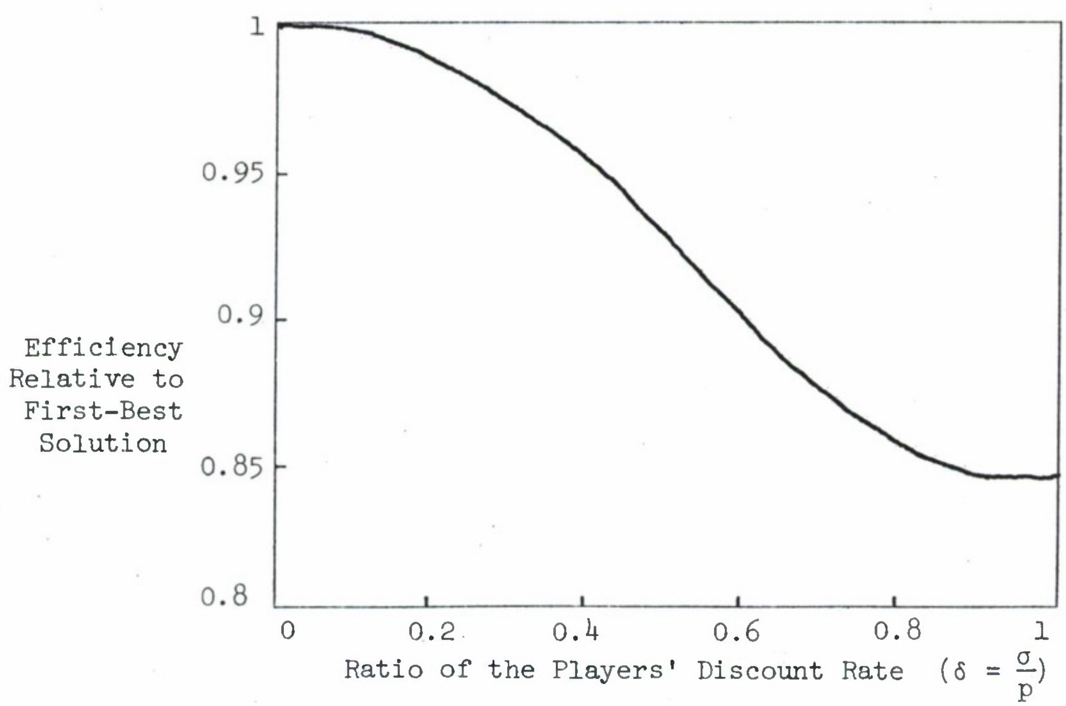


Figure 2

Efficiency as a function of the ratio of the players' discount rates.



$$\tfrac{1}{3}(1 + \mu + \mu^2)[\delta - \gamma(2 - \delta)] + \tfrac{1}{2}(1 + \mu)[\gamma(1 + 2\mu(1 - \delta)) - 2\delta\mu]$$

$$+ \delta\mu^2 - \gamma\mu(1 - \delta\mu) + \frac{\delta^{\gamma-1}(1 - \mu)^\gamma}{1 + \gamma}\left(2\delta\mu - 1 + 2(1 - \delta)\left(1 - \frac{1 - \mu}{2 + \gamma}\right)\right) = 0 \quad . \tag{R}$$

Given $\delta = \sigma/\rho$, a root μ $[0,1/2]$ to (R) is easily found numerically.

The sum of the players' ex ante utilities is computed as follows:

$$\bar{U} + \bar{V} = \int_\mu^1 \Big[\int_0^{\delta(b-\mu)} (b - s)ds + \int_{\delta(b-\mu)}^1 \Big[b\left(\frac{\delta(b - \mu)}{s}\right)^\gamma - s\left(\frac{\delta(b - \mu)}{s}\right)^{1+\gamma}\Big]ds\Big]db$$

$$= \int_\mu^1 [\delta(b - \mu)[(1 - \tfrac{1}{2}\delta)b + \tfrac{1}{2}\delta\mu]$$

$$+ \frac{1}{1 - \gamma}[(1 - \delta)b + \delta\mu](\delta(b - \mu))^\gamma(1 - [\delta(b - \mu)]^{1-\gamma})]db$$

$$= \int_\mu^1 \frac{\delta}{1 - \gamma} [[\tfrac{1}{2} \delta(1 + \gamma) - \gamma]b^2 + [\mu\gamma - \delta\mu(1 + \gamma)]b$$

$$+ \tfrac{1}{2} \delta\mu^2(1 + \gamma) + \delta^{\gamma-1}[(1 - \delta)b + \delta\mu](b - \mu)^\gamma]db$$

$$= \frac{\delta(1 - \mu)}{1 - \gamma} [\tfrac{1}{3}(1 + \mu + \mu^2)[\tfrac{1}{2}\delta(1 + \gamma) - \gamma] + \tfrac{1}{2}(1 + \mu)[\mu\gamma - \delta\mu(1 + \gamma)]$$

$$+ \tfrac{1}{2} \delta\mu^2(1 + \gamma) + \frac{\delta^{\gamma-1}(1 - \mu)^\gamma}{1 + \gamma}(\delta\mu + (1 - \delta) \frac{1 + \mu + \gamma}{2 + \gamma})] \quad .$$

The value of μ and the efficiency of the ex ante efficient mechanism relative to the first-best (full information) solution are shown in Figure 1 and Figure 2, respectively, as the ratio of the players' discount rates is varied from 0 to 1. Bargaining efficiency improves as the seller's discount rate is increased relative to the

buyer's. When the players' discount rates are equal, 15.6% of the gains from trade are lost due to delays in agreement. This inefficiency decreases to zero as $\rho \to \infty$, illustrating theorem 2.

6. Conclusion

Two important features of any bargaining setting are information and time. Bargainers typically have incomplete information about each other's preferences, and therefore must communicate some of their private information in order to determine whether or not gains from trade exist. One means of communication is for the agents to signal their private information through their willingness to delay agreement: bargainers that anticipate large gains from trade will be unwilling to delay agreement and so will propose attractive terms of trade that the other is likely to accept early in the bargaining process; whereas, bargainers expecting small gains will prefer to wait for better offers from their opponent. In this paper, I have described the properties of such a bargaining model, by analyzing a sequential direct revelation game.

Modeling the bargaining process as a sequential game, where the agents communicate their private information over time, has two main advantages. First, from the point of view of realism, one commonly observes bargaining taking place over time. Second, any static bargaining mechanism, because it does not permit the agents to learn about their opponent's preferences, must end with positive probability in a situation where gains from trade are possible and yet no agreement

is reached. If both bargainers know that gains from trade exist, what is preventing them from continuing negotiations until an agreement is reached? By introducing the time dimension, and hence allowing the bargainers to communicate through their actions over time, one is able to construct perfect bargaining mechanisms, in which the bargainers continue to negotiate so long as they expect positive gains from continuing.

When the bargainers discount future gains according to known and fixed discount rates, it was found that the bargainers may be better off (in terms of their ex ante utilities) using a sequential bargaining mechanism than a static scheme. This is because the time dimension introduces an additional asymmetry into the problem, which may be exploited in order to construct sequential bargaining mechanisms that ex ante dominate the most efficient static mechanisms. Even in situations where a static mechanism is ex ante efficient it is unlikely that such a mechanism would be adopted by the bargainers, since it necessarily would violate sequential rationality.

The analysis presented here represents an early step towards understanding how agreements are reached in conflict situations under uncertainty. Several simplifying assumptions have been made in order to keep the analysis manageable. First, modeling the agents' time-preferences with constant discount rates is an appealing example, but not an accurate description of all bargaining settings. [5] Second, the agents have been assumed to be risk neutral, but in many bargaining situations the agents' willingness to take risks is an important

bargaining factor. Third, I have restricted attention to rational agents who can calculate (at no cost) their optimal strategies. Certainly, few agents are so consistent and calculating. With less than rational agents, an agent's capacity to mislead his opponent becomes an important variable in determining how the gains from trade are divided. Finally, I have assumed that the players' valuations are independent. In many settings, the bargainer's valuations will be correlated so, for example, the seller's willingness to trade may be a signal of the valuation of the object to the buyer.

Although it would be useful in future research to weaken the simplifying assumptions made here, perhaps the most fruitful avenue for future research is in analyzing specific extensive-form bargaining games. The advantage of looking at specific extensive-form games is that for such games the bargaining rules are independent of the probabilistic beliefs that the players have about each other's preferences. In a direct revelation game, on the other hand, the bargaining rule depends in a complicated way on these probabilistic beliefs. Because of this dependence, direct revelation games are not played in practice.

Can one find a strategic game that comes close to implementing the ex ante efficient bargaining mechanism over a wide range of bargaining situations? Initial studies along these lines have been done by Fudenberg and Tirole [1983], Sobel and Takahashi [1983], and Cramton [1983]. All three papers consider a model in which only one of the bargainers makes offers. When the player's reservation prices are uniformly distributed on $[0,1]$ and their discount rates are equal, it

was found that this model resulted in 32% of the gains from trade being lost, as opposed to a 16% loss if the ex ante efficient bargaining mechanism was adopted (Cramton [1983]). Thus, the players inability to commit to ending negotiations results in a bargaining outcome that is significantly less efficient than if commitment were possible.

Perhaps a better candidate for a strategic bargaining game that is nearly ex ante efficient is the game in which the bargainers alternate offers. This game was analyzed by Rubinstein [1982] in a setting of complete information, but an analysis with incomplete information has yet to be done. Of particular interest is the alternating-offer game as the time between offers goes to zero, for this strategic game represents a very general bargaining rule: at any time a bargainer may make a new offer or accept the most recent offer of his opponent. It would be a pleasant surprise if such a reasonable bargaining game was nearly ex ante efficient over a variety of circumstances.

A second promising area for future research is further study on the implications of sequential rationality to bargaining and to more general games of incomplete information. I intend to address this issue in depth in a future research paper entitled "Perfect Bargaining Mechanisms."

Footnotes

1/ I hae assumed arbitrarily that the seller is female and the buyer is male.

2/ Note that P is absolutely continuous, since U and S are absolutely continuous (they can be represented by indefinite integrals by (S)); thus, the fundamental theorem of integral calculus applies.

3/ This is often referred to as "full-information efficiency" in the literature. Holmstrom and Myerson [1981] call this "ex post classical efficiency" to distinguish it from their concept of ex post incentive-efficiency, in which incentive constraints are recognized.

4/ One might think that the assumption of equal weights is made without loss of generality, because the payoff functions here are additively separable in money and goods, and thus utility is transferable between players. Although this intuition is correct in a setting of complete information, it is false when there is incomplete information, because an ex ante transfer of utility will violate individual rationality for some players.

5/ Fishburn and Rubinstein [1982] derive when the discounting assumption is valid. In particular, they prove that any preferences over bargaining outcomes that are monotonic, continuous, and stationary can be represented by discounting provided the bargainers exhibit impatience over all outcomes except that of no agreement.

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Paper II: Bargaining With Incomplete Information
An Infinite-Horizon Model With Continuous Uncertainty

1. Introduction

Embedded within nearly every transaction is a bargaining problem: How should the gains from trade be split among the parties involved in the transaction? In a setting of perfect competition, the answer is simple: price is set so that the marginal seller and buyer reap no gains from trade. But in the absence of perfect competition, determining how to split the pie is a difficult question that has long been a concern of economists. Edgeworth, in fact, considered bargaining to be the fundamental problem of economics. Of course he said this at a time when markets were far less established, and fixing pricing was less prevalent. But today bargaining is just as important.

Two approaches in economics have been taken in analyzing the bargaining problem. The first is the axiomatic or cooperative approach, which focuses exclusively on the bargaining outcome rather than on the process of bargaining. In the axiomatic theory, a number of assumptions are made that restrict the bargaining outcome to a unique solution from among the set of possible agreements. It is generally assumed that each party has perfect information about the other's preferences, and that an efficient solution will be reached without delay. The second method, and the one adopted in this paper, is the strategic approach, which models the parties' negotiating behavior explicitly as moves in a non-cooperative game. Each party employs a bargaining strategy based on his or her beliefs about the other's strategy.

It is my hypothesis that in any real-life bargaining setting the relative urgency of the parties to reach agreement, the information each party has about the other's preferences, and the parties' ability to commit to particular strategies are all important determinants of the bargaining outcome. A producer with superior knowledge of a consumer's preferences may be able to exploit this knowledge to obtain a higher price for the product. Similarly, a producer supplying a customer who is in desperate need of a product may be able to get a higher price because of the customer's reluctance to delay agreement. The objective of this paper is to explore how information, time, and commitment affect the bargaining outcome. Of particular interest are questions of efficiency and distribution:

- What are the sources of bargaining inefficiencies?
- What determines how the gains from trade are split among those involved in the transaction?

In order to answer these questions, I develop a model of the bargaining process. In the basic model, two parties, a buyer and a seller, are bargaining over the price of an object. As they bargain, their payoffs are discounted over time, so that both the buyer and the seller have an incentive to come to an early agreement. The process is complicated by the fact that each agent may have incomplete information about the preferences of the other agent. In particular, the seller may not know how much the buyer values the object and the buyer may be unaware what it will cost the seller to acquire the object. The

sequential nature of the bargaining process combined with uncertainty over preferences means that communication between the agents is an important aspect of their behavior. The seller, when making an offer, must evaluate how the offer will reveal information to the buyer. Likewise, the buyer must interpret an offer as a signal of the seller's preferences and hence an indication of what to expect in subsequent rounds of the bargaining process.

Although I have described the model in terms of a buyer and a seller negotiating over the price of an object, the model applies to a much broader class of conflict situations: court settlements between plaintiff and defendant, contract negotiations between labor and management, trade agreements between nation states, and so on.

My approach is to model this bargaining process as a sequential game with incomplete information. The bargaining game is one of incomplete information, since one or both of the bargainers has private information unknown to the other. Rationality is assumed by requiring that the bargaining strategies of the agents form a Bayesian Nash equilibrium (Harsanyi [1967]): each player's strategy must be a best response to the other's strategy given their probabilistic beliefs of the state of the world. I further require that their behavior be sequentially rational (Kreps and Wilson [1982b]): at any stage of the game, the players must play optimally, given their beliefs, for the remainder of the game. Thus, players are unable to commit to strategies they would not wish to carry out. For example, the seller cannot threaten to raise the price should the buyer reject, if it is in the

seller's best interest to lower the price in the event that the buyer rejects the offer. A Nash equilibrium that satisfies sequential rationality is said to be a <u>sequential equilibrium</u>.

In situations where bargainers are unable to make binding commitments, it is unrealistic to end the bargaining exogenously after any finite number of periods - the bargaining should continue so long as the bargainers expect positive gains from continuing. Thus, even when the players are better off (ex ante) restricting negotiations to a finite number of periods, if they are unable to commit to walking away from the bargaining table, then they must adopt strategies that assume negotiations could potentially continue indefinitely. To allow the bargainers to cut off negotiations when there are positive gains from continuing would violate sequential rationality, since the players would be better off continuing and receiving a positive expected gain than ending negotiations and receiving a payoff of zero.

<u>Related Research</u>. Much has been written on bargaining. The work, both applied and theoretical, spans several disciplines, most notably labor relations, economics, psychology, and law. A discussion of all the approaches to the bargaining problem is not possible here. Instead, I will limit my discussion to the three strands of current economic research upon which my work is based: sequential bargaining, strategic information transmission, and bilateral trading.

Its closest connection is with the sequential bargaining literature, which models the bargaining process as a noncooperative game in which a sequence of offers is made over time. As time passes, delay

costs are incurred by both players, thus providing an incentive to reach an early agreement. Many of the papers on sequential bargaining examine the game in which the players have complete information about each other's preferences. Rubinstein [1982] considers the problem of how a fixed pie is split between two fully informed players. Players alternate making offers until an offer is accepted. When the players' payoffs are discounted over time, the game has a unique sequentially rational equilibrium in which trade occurs in the first period. This efficient bargaining outcome is due to the assumption of complete information: the players, being fully informed, are able to unravel what would happen in the course of the game, and thus are prepared to make and accept a reasonable initial offer and thereby avoid any costs of delay. Binmore [1980, 1982], McLennan [1982a], and Mori [1982] generalize the Rubinstein model to bargaining over a set of possible outcomes. In addition, they show that as the time between offers goes to zero (so that the person making the initial offer no longer has an advantage), the bargaining outcome approximates the Nash solution of the axiomatic theory, thus providing a noncooperative justification for the Nash solution.

In all these models, the following result typically holds: complete information implies an efficient bargaining outcome. One explanation for the common occurrence of inefficient outcomes (strikes, wars, costly delays) is that bargaining rarely occurs in an environment of complete information. Players will typically use the early rounds of the bargaining game to communicate their preferences to their

opponent. For example, a seller with high costs will try to persuade or signal to the buyer that he has high costs by making higher offers than he would if he had lower costs.

The papers of Binmore [1981], Cramton [1983b], Fudenberg and Tirole [1983], Fudenberg, Levine and Tirole [1983], Sobel and Takahashi [1983], Perry [1982], and Rubinstein [1983] analyze both the incomplete information and sequential aspects of bargaining games, and thus are most closely related to my work. Fudenberg and Tirole characterize rational behavior of agents in a two-period model when there are two potential types of buyers and two potential types of sellers. Sobel and Takahashi focus mainly on an infinite-period model with one-sided uncertainty. I have freely borrowed from their insightful work in my analysis of this problem in section 4. Fudenberg, Levine, and Tirole continue the analysis of infinite-horizon bargaining with one-sided uncertainty. Rubinstein analyzes an infinite-horizon game with alternating offers in which there is one-sided uncertainty about a player's delay cost. Binmore explores the validity of the generalized Nash bargaining solution by comparing it with the outcome of a non-cooperative game with incomplete information. Perry considers an infinite-horizon game with two-sided uncertainty in which the players have fixed transaction costs of making offers. In contrast to the models with complete information, it is found in these models that incomplete information leads to inefficient bargaining outcomes. In all but Perry's model, agreement is delayed as a result of incomplete information. The players are able to communicate their private

formation through their willingness to delay aggrement. No such communication is possible in Perry's model, since every type of each player has the same willingness to delay agreement. As in the Spence signalling model [1974], a requirement for there to be communication through delayed aggreement is that delay be more costly to those who expect large gains from trade. It seems plausible that this requirement would be met in most situations. Cramton [1983b] considers a sequential direct revelation game and characterizes the class of perfect bargaining mechanisms, which are incentive compatible, individually rational, and sequentially rational.

Sequential games with incomplete information have also been analyzed in areas other than bargaining. In the industrial organization literature, several pioneering studies have sought to explain oligopolistic behavior that cannot be accounted for in a world of complete information (Kreps-Wilson [1982a]; Kreps-Milgrom-Roberts-Wilson [1982]; Milgrom-Roberts [1982a], [1982b]; Saloner [1982]). Closely related to these studies are models of information transmission, in which agents with divergent interests transfer information prior to making a decision (Crawford-Sobel [1981]; Green-Stokey [1981]; McLennan [1982]).

Finally, there are a number of bilateral trading models that ignore the sequential aspects of bargaining, concentrating on incomplete information alone (Chatterjee [1982]; Chatterjee-Samuelson [1981]; Green-Honkapohja [1981]; Myerson [1979]; and Myerson-Satterthwaite [1983]). These studies focus on the choice of an efficient mechanism for

resolving the conflict among traders. A weakness of these papers is that by ignoring the time dimension the bargainers are unable to learn through past actions whether or not gains from trade exist. Hence, trade will frequently not occur even in situations where trade is profitable.

Outline.

Here I analyze an infinite-horizon model with two-sided uncertainty in which there is a continuum of potential buyers and sellers. My main interest is in the effects of incomplete information on the behavior of the agents; thus, most of the paper is spent analyzing the bargaining game in which each agent is unsure of the preferences of the other agent. Equilibrium behavior is also determined for the game with one-sided incomplete information. Although this game is of less general interest, due to the less realistic informational assumption, its analysis provides a convenient stepping stone to the more intricate case of two-sided uncertainty. In Section 6, I examine how information, uncertainty, and timing influence the bargaining behavior of rational agents. The fundamental results are that:

- Incomplete information leads to bargaining inefficiency.
- Inefficiencies increase as preferences become more uncertain.
- Bargainers with high delay costs are at a disadvantage.
- Information is revealed more quickly, the higher the delay costs.

Although these intuitive results are all derived for the special case in which the potential gains from trade are uniformly distributed, many of the results are true in more general settings as well. For

example, the conclusion that incomplete information leads to bargaining inefficiency has been observed in numerous economic models with incomplete information. On the other hand, the result that bargainers with high delay costs are at a disadvantage does not hold true for all distributions of the gains from trade. Fudenberg and Tirole [1981] have shown that at least in a two-period model it is possible for a player to benefit from a high delay cost. Perhaps the most novel result is the form of the equilibrium I derive, in which information is revealed gradually ove time and the rate of revelation depends on the players' costs of delay. In previous bargaining models with incomplete information, the communication process was either not modeled (as in the static models) or the learning was cut short due to a restricted bargaining horizon (as in the two-period models).

2. The Model

Two parties, a buyer and a seller, are bargaining over the price of an object which can be produced by the seller at a cost s and is worth b to buyer.[1/] The seller's cost s and the buyer's valuation b are also referred to as their reservation prices, since they represent respectively the minimum and maximum price at which each would agree to trade. At every stage of the game, the seller makes an offer p, whch the buyer may accept or reject. should the buyer prompts the seller to make another offer in the next stage of the game. Both the buyer and the seller have costs of delaying the bargaining process. Specifically, their payoffs in the subsequent rounds are discounted

according to the discount factors δ_b for the buyer and δ_s for the seller, with $0 < \delta_b, \delta_s < 1$. Thus the payoffs, if the buyer accepts the n-th offer p, are $\delta_b^{n-1}(b - p)$ for the buyer and $\delta_s^{n-1}(p - s)$ for the seller. Should they fail to reach agreement both players' payoffs are zero.

The buyer, though aware of his own valuation b, does not know the seller's cost of production s, but assesses her cost to be distributed according to the distribution $F(s)$, with a positive density $f(s)$ on $[\underline{s},\bar{s}]$. Similarly, the seller knows her cost s, but can only assess the buyer's valuation to be distributed according to the distribution $G(b)$, with a positive density $g(b)$ on $[\underline{b},\bar{b}]$. The discount factors, the distributions of the potential buyers and sellers, and the structure of the game are common knowledge. In addition, it is assumed that both the buyer and the seller are solely interested in maximizing their expected monetary gain.

Throughout this paper I deal primarily with the example in which the seller's cost is distributed uniformly on the interval $[\underline{s},\bar{s}]$, and the buyer's valuation is distributed uniformly on $[\underline{b},\bar{b}]$ and is independent of the seller's cost. It is natural to assume that $\bar{s} < \bar{b}$ and $\underline{s} < \underline{b}$, for a seller with $s > \bar{b}$ or a buyer with $b < \underline{s}$ would have no hope of gaining anything from trade. It will be shown that a cutoff strategy, in which the buyer accepts p_t if and only if his valuation b exceeds an indifference valuation $\hat{b}_t(p_t)$, is optimal for the buyer in every period t.[2/] The buyer's use of a cutoff strategy together with the fact that a truncated, uniformly distributed random variable is

still uniformly distributed implies that the seller's problem takes on a simple form: at each stage, the seller selects a price given that the buyer's valuation is uniformly distributed on the interval $[\underline{b},\hat{b}]$ where $\hat{b}$ is the most recent indifference valuation. This results in a stationary solution to the seller's dynamic programming problem, making the problem tractable.

I derive an equilibrium in which the bargainers reveal gradually their private information over time. At each stage of the game, the buyer accepts the seller's offer p if his valuation is greater than some cutoff valuation $\hat{b}(p)$. Thus, a rejection by the buyer indicates to the seller that the buyer's valuation is less than $\hat{b}(p)$. Similarly, the extreme low-cost sellers (sellers with costs less than $\hat{s}$) make an offer $p(s)$, which completely reveals their information; whereas, high-cost sellers $(s > \hat{s})$ pool together by making an offer that no buyer will accept.

In sequential games with incomplete information, players typically have an incentive to hide their private information. Thus, the seller would like to tell the buyer, "My costs are high, so you better expect to pay a high price," regardless of whether or not the seller's costs are in fact high. The buyer, of course, is aware of the seller's incentive to deceive and hence will not believe statements that are not backed up by actions. The seller, in order to convince the buyer that she has high costs, must take actions that a low-cost seller would be unwilling to take (as in a signalling problem such as Spence [1974]). Likewise, a low-cost seller must take actions that no high-cost seller

would find attractive. Intuitively, this is why the seller reveals her private information in the way she does. Since low-cost sellers are unwilling to delay agreement by making high offers, a high-cost seller signals that her costs are high by making high offers. As the seller's cost increases, she will make higher and higher offers, which are accepted by fewer and fewer buyers. At some point $(s = \hat{s})$, the seller makes an offer that no buyer will accept, since every buyer is better off waiting for lower prices in the future. All sellers with costs $s > \hat{s}$ are then unable to reveal their private information in the current round: a seller with cost $s' > \hat{s}$ cannot convince the buyer that she has a cost s', since she has no way to back up her statement with actions that other sellers with costs $s > \hat{s}$ would be unwilling to take. These high-cost sellers must pool together, revealing only that their costs are greater than $\hat{s}$.

Exactly how much information is revealed in each round of negotiations will depend on the bargainers' costs of delay. If delay costs are high, more information will be revealed because the punishment to low-cost sellers of pretending to have higher costs is greater; whereas, if the seller's delay cost is low (δ_s close to one), then less information will be revealed by the seller, since she is more willing to delay agreement by offering higher prices.

3. General Characterization of Equilibria

A sequential equqilibrium consists of functions that determine the players' optimal strategies given their information about how the game

has evolved for each information set, including information sets off the equilibrium path. The seller beings by choosing an optimal price schedule $p_o(s)$ given her cost s and her belief that the buyer's valuation is distributed on $[\underline{b},\bar{b}]$ with density g.[3/] Next, the buyer decides to accept or reject the offer given the initial price p and his own valuation b; i.e., the buyer chooses a binary function $a_o(p,b) \in \{accept, reject\}$. This strategy can be simplified to an indifference valuation $\hat{b}_o(p)$,since a cutoff strategy in which a buyer accepts the offer if and only if $b > \hat{b}_o(p)$ is optimal for the buyer (as deerived in Theorem 1 below). When determining whether or not to accept or reject the offer p, the buyer must make an inference about the seller's cost: what informatin does the offer p reveal about the seller's cost? This inference then determines what prices the buyer expects in the future, which enables him to calculate whether he should accept the price p now or wait for lower prices in the future. Should the buyer observe a price p in the range of the equilibrium price schedule $p_o(s)$, then the buyer will update his prior belief of the seller's cost using Bayes' rule. If, however, the buyer observes a price p not in the range of the price schedule $p_o(s)$, then he cannot use Bayes' rule to update his prior. Rather he must update his prior based on conjectures he has about the seller's cost when he is surprised by nonequilibrium behavior. These conjectures are needed to determine the buyer's best response to behavior off the equilibrium path, which in turn is used to evaluate the seller's benefits from deviating from the equilibrium. Of course, in equilibrium, the seller is better off

offering $p_o(s)$ than deviating, since the seller's price schedule is a best response to the buyer's strategy.

At every stage of the game, the strategies are similar: the seller chooses an optimal price schedule p_t given the history of events, and the buyer maintains a set of conjectures μ_t that determine his beliefs about the seller's cost should he observe nonequilibrium behavior. No such conjectures are necessary for the seller since in equilibrium both possible actions for the buyer (accept or reject) occur with positive probability: the seller is never surprised by observing an event with prior probability zero. A sequential equilibrium, then, in the infinite-horizon bargaining game with two-sided uncertainty is the collection

$$\{p_t(\cdot),\hat{b}_t(\cdot),\mu_t(\cdot)\}_{t=0}^{\infty}$$

where $\mu_t[s|p_t \in p_t([\underline{s},\bar{s}])]$ is a probability distribution representing the buyer's conjectures about the seller's cost, conditioned on the event that the seller offered a price p_t that is not in the range of the equilibrium price schedule $p_t(s)$. The equilibrium strategies p_t and $\hat{b}_t$ and the conjectures μ_t must be such that at time t:

(1) the buyer $\hat{b}_t(p)$ is indifferent between accepting or rejecting the price p given his expectations of future prices based on his inference of the seller's cost,

(2) the offer $p_t(s)$ of seller s is optimal for the seller given that the buyer's valuation is distributed according to the distribution $G(b)$ with support $[\underline{b},\hat{b}_t]$, and

(3) the conjectures μ_t imply that the seller s is better off offering the equilibrium price $p_t(s)$ than deviating by offering a price not in the range of the equilibrium price schedule p_t.

I now summarize a number of necessary conditions that every sequential equilibrium in the infinite-horizon bargaining game must satisfy. These properties are quite general, depending only on the fact that the distributions of the players' reservation prices have positive densities on an interval. The first condition provides a strong characterization of the players' expected payoffs. This result was first derived in Myerson and Satterthwaite [1983]. I present their proof for completeness.

Let $U(s)$ be the expected payoff to the seller given that her cost is s and let $V(b)$ be the expected payoff to the buyer that given his valuation is b. It is convenient to split each player's equilibrium payoff into two components: the expected cost and the expected benefit of the transaction. Thus, let $U(s) = S(s) - sP(s)$ and $V(b) = bQ(b) - B(b)$ where

$$S(s) = \sum_{t=0}^{\infty} \delta_s^t p_t(s) \Pr\{p_t \text{ accepted}\} \qquad P(s) = \sum_{t=0}^{\infty} \delta_s^t \Pr\{p_t \text{ accepted}\} .$$

which may be interpreted as the expected payment to the seller and the discounted probability of agreement, respectively. $Q(b)$ and $B(b)$ are defined in the same fashion.

<u>Theorem 1</u>: Every sequential equilibrium in the infinite-horizon bargaining game with two-sided uncertainty has the following

characteristics:

(1) The seller's expected payoff U is convex and decreasing, with derivative $dU/ds = -P$ almost everywhere on $[\underline{s},\bar{s}]$, her discounted probability of agreement P is decreasing, and

$$U(s) - U(\bar{s}) = \int_s^{\bar{s}} P(u)du \qquad S(s) - S(\bar{s}) = \int_s^{\bar{s}} -udP(u) \quad . \qquad (S)$$

Similarly, the buyer's expected payoff V is convex and increasing, with derivative $dV/db = Q$ almost everywhere on $[\underline{b},\bar{b}]$, his discounted probability of agreement Q is increasing, and

$$V(b) - V(\underline{b}) = \int_{\underline{b}}^{b} Q(u)du \qquad B(b) - B(\underline{b}) = \int_{\underline{b}}^{b} udQ(u) \quad .$$

(2) At every stage t, the buyer employes a cutoff strategy in which he accepts an offer p_t if and only if his valuation b is less than some cutoff valuation $\hat{b}_t(p_t)$. Thus, the seller's posterior belief at time t of the buyer's valuation is $G(b)/G(\hat{b}_t)$.

(3) Expected price, conditional on agreement, decline over time; that is, for all buyers that reject p_t,

$$p_t > E(p_{t+\tau}|\text{agreememt at } t + \tau) \quad \tau > 0.$$

<u>Proof</u>:

(1) By definition, seller s achieves the payoff $U(s) = S(s) - sP(s)$. Alternatively, seller s can pretend to be seller s' in which case her payoff is $S(s') - sP(s')$. In equilibrium, seller s must not want to pretend to be seller s', so we have $U(s) \geq S(s') - sP(s')$ for

all $s, s' \in [\underline{s},\bar{s}]$, or

$$U(s) \geq U(s') - (s - s')P(s')$$

implying that U has a supporting hyperplane at s' with slope $-P(s') \leq 0$. Thus U is convex and decreasing with derivative $(dU/ds)(s) = -P(s)$ almost everywhere and P must be decreasing. Since P is monotone it is differentiable almost everywhere and we have that $(dS/ds)(s) = s(dP/ds)(s)$, which yields (S). The proof for the buyer is identical.

(2) Define $V(b,H_t)$ to be the equilibrium expected payoff at time $t + 1$ of a buyer with valuation b after a history H_t, and let $Q(b,H_t)$ be the discounted probability of trade for the buyer b after a history H_t. Suppose a buyer with valuation b chooses to accept the offer p_t. Then $b - p_t \geq \delta_b V(b,H_t)$. Now consider a buyer with valuation $b' > b$. We wish to show that the buyer b' will prefer to accept p_t; that is, $b' - p_t \geq \delta_b V(b',H_t)$. Buyer b can follow the equilibrium strategy of buyer b', but it must be the case that buyer b does at least as well by following his own equilibrium strategy than that of buyer b'. Thus,

$$V(b,H_t) \geq V(b',H_t) - (b' - b)Q(b',H_t) \geq V(b',H_t) + b - b' \quad ,$$

and so

$$b - p_t \geq \delta_b[V(b',H_t) + b - b'] \quad ,$$

or

$$\delta_b b' + (1 - \delta_b)b - p_t > \delta_b V(b',H_t) \quad .$$

Since $b' > \delta_b b' + (1 - \delta_b)b$, this yields

$$b' - p_t > \delta_b V(b',H_t) \quad ,$$

which implies that any buyer with valuation greater than b strictly prefers to accept p_t now, rather than wait for future offers.

(3) Suppose buyer b rejects an offer of p, so that the bargaining continues. Then it must be that buyer b expects to do better by waiting; that is, he must expect to accept a price sufficiently less than p so as to compensate him for waiting. Since expectations are confirmed in equilibrium, it must be the case that expected price decline over time. (Note that this argument does not imply that every bargaining realization has prices declining over time, only that on average prices decline. It is not inconceivable that the sellers could pool together in the first round at a price p and then separate in the next period in such a way that the highest cost seller offers a price greater than p.) Q.E.D.

A final property that is very reasonable, but which cannot be proven in general, is that a seller with higher costs offers higher prices. This is certainly true in the one-shot game, but in the multi-period game the buyer might have strange beliefs that could sustain an equilibrium in which price does strictly decrease with s in some period. For example, the buyer might believe in the initial period that a high offer signals a low-cost seller. Such a belief could in fact

entice the low-cost seller to make higher offers in the initial period, but eventually the low-cost seller must make lower offers than a seller with higher costs. However, I do not allow the buyer to maintain such beliefs, and so restrict attention to monotonic equilibria.

4. Equilibrium Behavior with the Seller's Cost Known

In periods following the seller's revelation of her private information, the players' behavior will be the same as in the game in which the buyer knows the seller's cost s, and the seller knows only that the buyer's valuation is uniformly distributed on $[\underline{b},\hat{b}(p)]$. Thus, to determine a separating equilibrium for the game with two-sided uncertainty, it is necessary to first determine an equilibrium in the game with one-sided uncertainty.

Assume that the seller's cost s is known to the buyer ($\underline{s} = \bar{s}$), but the seller only knows that the buyer's valuation is uniformly distributed on $[\underline{b},\bar{b}]$ (without loss of generality, we can assume that $s \leq \underline{b}$, for any buyers with $b < s$ would not enter negotiations). I begin by determining the equilibrium for the n-stage game, and then establish an equilibrium in the infinite-horizon game by letting n go to infinity. It will turn out that the form of the infinite-horizon bargaining equilibrium will depend on whether $s = \underline{b}$ or $s < \underline{b}$. With $s = \underline{b}$, prices strictly decrease over time and eventually converge to (but never reach) $\underline{b}$. With $s < \underline{b}$, after some finite number of periods, the seller offers the price $p = \underline{b}$, which is accepted by the buyer with probability one, thus concluding the bargaining.

First consider the case with $s = \underline{b}$, so the seller will never offer the price $p = \underline{b}$. The players' equilibrium behavior (a sequence of prices for the seller and a sequence of indifference valuations for the buyer) is determined by solving a dynamic programming problem in which the seller chooses the offer that maximizes her present value of current and future gains, given her knowledge of the buyer's valuation, and subject to the constraint that the buyer will accept the offer only if his valuation is sufficiently high that he is better off accepting now than waiting for lower prices in the future. Namely, with i periods remaining in the n-stage bargaining game, define j to be $n + 1 - i$, so the seller chooses p_j to maximize her expected gain $u_j(s,b_{j-1})$ given that the buyer's valuation is uniformly distributed on $[\underline{b},b_{j-1}]$:

$$u_j(s,b_{j-1}) = \max_p \frac{1}{b_{j-1} - \underline{b}} [(p - s)(b_{j-1} - b_j) + \delta_s(b_j - \underline{b})u_{j+1}(s,b_j)]$$

$$\text{such that} \quad b_j - p = \delta_b(b_j - p_{j+1})$$

<u>Theorem 2 (Sobel and Takahashi)[4/]</u>: When $s = \underline{b}$ and the buyer's valuation is uniformly distributed on $[\underline{b},b_n]$, the n-stage bargaining game with one-sided uncertainty has a unique sequential equilibrium with the seller's expected profit $u_j(s,b_{j-1})$ and price $p_j(s,b_{j-1})$ with i periods remaining and $j = n + 1 - i$ given by

$$u_j(s,b_{j-1}) = \frac{1}{2} c_j \frac{(b_{j-1} - s)^2}{b_{j-1} - \underline{b}}$$

$$p_j(s,b_{j-1}) = c_j(b_{j-1} - s) + s$$

where $c_n = 1/2$ and for $i > 1$

$$c_j = \frac{(1 - \delta_b + \delta_b c_{j+1})^2}{2(1 - \delta_b + \delta_b c_{j-1}) - \delta_s c_{j+1}}$$

Moreover, the buyer's indifference valuation $b_j(s,b_{j-1})$ with $i - 1$ periods remaining is given by

$$b_j(s,b_{j-1}) = \frac{1 - \delta_b + \delta_b c_{j+1}}{2(1 - \delta_b + \delta_b c_{j+1}) - \delta_s c_{j+1}} (b_{j-1} - s) + s \quad .$$

Proof: The proof is by induction on n. With one period remaining, the seller wishes to choose p according to the program.

$$u_n(s,b_{n-1}) = \max_p \, (p - s) \frac{b_{n-1} - p}{b_{n-1} - \underline{b}}$$

so $p_n(s,b_{n-1}) = (1/2)(b_{n-1} + s) = (1/2)(b_{n-1} - s) + s$ and $u_n(s,b_{n-1}) = (1/4)(b_{n-1} - s)^2/(b_{n-1} - \underline{b})$.

With i periods remaining, the seller's expected profit is given by

$$u_j(s,b_{j-1}) = \max_p \frac{1}{b_{j-1} - \underline{b}} [(p - s)(b_{j-1} - b_j) + \delta_s(b_j - \underline{b})u_{j+1}(s,b_j)] \tag{1}$$

$$\text{such that } b_j - p = \delta_b(b_j - p_{j+1})$$

Assume by the induction hypothesis that

$$u_{j+1}(s,b_j) = \frac{1}{2} c_{j+1} \frac{(b_j - s)^2}{b_j - \underline{b}}$$

$$p_{j+1}(s,b_j) = c_{j+1}(b_j - s) + s$$

Then $p = (1 - \delta_b)b_j + \delta_b[c_{j+1}(b_j - s) + s]$, or

(2) $$p = (1 - \delta_b + \delta_b c_{j+1})(b_j - s) + s$$

Substituting into (1) yields

(3) $$u_j(s,b_{j-1}) = \max_{b_j} \frac{1}{b_{j-1} - \underline{b}} [(1 - \delta_b + \delta_b c_{j+1})(b_j - s)(b_{j-1} - b_j) + \frac{1}{2} \delta_s c_{j+1}(b_j - s)^2]$$

which has a unique maximum[5/] when

$$[2(1 - \delta_b + \delta_b c_{j+1}) - \delta_s c_{j+1}]b_j = (1 - \delta_b + \delta_b c_{j+1})(b_{j-1} + s) - \delta_s c_{j+1} s$$

so,

(4) $$b_j = \frac{1 - \delta_b + \delta_b c_{j+1}}{2(1 - \delta_b + \delta_b c_{j+1}) - \delta_s c_{j+1}} (b_{j-1} - s) + s \quad .$$

Thus by substituting (4) into (2) and (3), we get

$$p_j(s,b_{j-1}) = \frac{(1 - \delta_b + \delta_b c_{j+1})^2}{2(1 - \delta_b + \delta_b c_{j+1}) - \delta_s c_{j+1}} (b_{j-1} - s) + s \quad , \text{ and}$$

$$u_j(s,b_{j-1}) = \frac{1}{2} \frac{(1 - \delta_b + \delta_b c_{j+1})^2}{2(1 - \delta_b + \delta_b c_{j+1}) - \delta_s c_{j+1}} \frac{(b_{j-1} - s)^2}{b_{j-1} - \underline{b}}$$

as required. Q.E.D.

Equilibrium behavior in the infinite-horizon model is derived as an immediate consequence of Theorem 2 by letting n go to infinity.[6]

Corollary: When $s = \underline{b}$ and the buyer's valuation is uniformly distributed on $[\underline{b},\bar{b}]$, then the seller's equilibrium price $p_i(s,\bar{b})$ in period i, her expected profit $u(s,\bar{b})$, and the buyer's indifference valuation b_{i-1} in period i are given by

$$p_i = c(\bar{b} - s)d^{i-1} + s$$

$$b_{i-1} = (\bar{b} - s)d^{i-1} + s$$

$$u = \frac{1}{2} c \frac{(\bar{b} - s)^2}{\bar{b} - \underline{b}}$$

where $d = c/(1 - \delta_b + \delta_b c)$ and $c(\delta_s,\delta_b)$ is defined implicitly by

$$c = \frac{(1 - \delta_b + \delta_b c)^2}{2(1 - \delta_b + \delta_b c) - \delta_s c} \; .$$

The equations for c and d above can be solved simultaneously to yield

$$d = \frac{1}{\delta_s}(1 - \sqrt{1 - \delta_s}) \qquad c = \frac{d(1 - \delta_b)}{1 - \delta_b d}$$

(It is easy to see that $0 < c, d < 1$ whenever $0 < \delta_b, \delta_s < 1$).

Now assume that $s < \underline{b}$ so that for sufficiently large n, the seller will at some point want to offer a price $p = \underline{b}$ that will be accepted with probability one. Then in the last period we have $p_n = \underline{b}$

and $u_n(s,b_{n-1}) = \underline{b} - s$. The equilibrium behavior in this case is derived as before by induction.

Theorem 3: For the n-stage game with $s < \underline{b}$ in which the seller ends the bargaining by offering $p_n = \underline{b}$, the seller's price $p_j(s,b_{j-1})$, her expected profit $u_j(s,b_{j-1})$, and the buyer's indifference valuation $b_j(s,b_{j-1})$ with i periods remaining and $j = n + 1 - i$ are given by

$$b_j(s,b_{j-1}) = x_j b_{j-1} + y_j s + z_j$$

$$p_j(s,b_{j-1}0 = c_j b_{j-1} + d_j s + e_j$$

$$u_j(s,b_{j-1}) = \frac{1}{b_{j-1} - \underline{b}} [\alpha_j b_{j-1}^2 + (\beta_j s + \gamma_j)b_{j-1} + \rho_j s^2 + \sigma_j s + \tau_j]$$

where x_j, y_j, z_j, c_j, d_j, e_j, α_j, β_j, γ_j, ρ_j, σ_j, and τ_j are constants depending on $\underline{b}$, δ_s, and δ_b.

Proof: The proof is analogous to that of Theorem 2, but much more tedious and so is done in the appendix.

5. Equilibrium Behavior with Two-Sided Uncertainty

With two-sided uncertainty, the seller must be concerned with the information her offers reveal to the buyer, and the buyer must carefully interpret offers as indications of the seller's true cost. Here I will focus on a separating equilibrium over time in which at each stage, low-cost sellers $(s < \hat{s})$ reveal completely their cost, while high-cost sellers pool together by offering a price so high that no buyer will

accept their offer. The equilibrium is monotonic in that sellers with higher costs offer higher prices: $p(s)$ is strictly increasing for $s < \hat{s}$ and constant for $s > \hat{s}$. Other equilibria are possible, such as a partition equilibrium in which $p(s)$ is a step function, but such partially revealing equilibria are intractable in the infinite-horizon game. Moroever, analysis of equilibria in the two-period model (Cramton [1983a]) suggests that the players have very little to gain by only revealing partially their information (it was found that the seller could typically increase her payoff by no more than one-tenth of one percent when she played the optimal partition strategy).

For those sellers that reveal completely their private information, their price schedule $p(s)$ is strictly increasing in s, so that the buyer is able to infer the seller's cost by inverting $p(s)$; namely, $s = p^{-1}(p)$. Thus, the players' strategies for the remainder of the game will be as determined in the previous section where the seller's cost is known. However, to insure incentive compatitility one must give seller s the option of pretending to be some other seller s' if she so desires. Suppose at some stage of the bargaining the seller knows she is facing a buyer whose valuation is uniformly distributed on $[\underline{b},\bar{b}]$ and she chooses to reveal (perhaps falsely) that her cost is s'. Seller s will choose s' and $p = p_o$ so as to maximize her expected gain given that the buyer infers her cost to be $s'(p)$ and accepts if $b > \hat{b}(p) = b_o$:

$$\max_{p,s'} (p - s)(\bar{b} - \hat{b}) + \sum_{n=1}^{\infty} \delta_s^n (p_n - s)(b_{n-1} - b_n)$$

subject to

(1) Sequentional Rationality. The seller's future offers $p_1, p_2, \ldots$ are chosen to maximize the payoff of seller s' given the buyer's future indifference valuations $b_1, b_2, \ldots,$ which are chosen so that buyer b_n is indifferent between accepting p_n now or waiting one period and accepting p_{n+1} next period:

$$b_n - p_n = \delta_b [b_n - p_{n+1}(s', b_n)] \quad \forall \; n > 0 \; .$$

(2) Incentive Compatibility. The buyer in equilibrium is not fooled:

$$s'(\cdot) = p^{-1}(\cdot)$$

A few comments are in order. First, the above optimization problem applies only for the range of sellers $[\underline{s}, \hat{s}]$ that reveal completely their information (so that inversion of $p(s)$ is possible). Second, the problem as stated only allows for deviations along the equilibrium path; that is, it is initially assumed that a seller s will imitate the behavior of seller s' forever and hence never be detected as deviating from the equilibrium. Deviations off the equilibrium path are considered later in this section, when I establish conjectures that support the described equilibrium strategies. For example, a seller s may wish to pretend to be a seller with cost s' for three periods and then to be a seller with cost s'' for two more periods and then act like herself for the remainder of the game. Actually, sequential rationality prequires that the seller's future

offers $p_1, p_2, \ldots$ be chosen to maximize the utility of seller s, not s'. However, since the buyer believes that the seller has cost s' and so expects to see prices that maximize the utility of seller s', if the buyer observes prices that do not maximize the utility of seller s' then his subsequent behavior will be determined by his conjectures off the equilibrium path. I have chosen these conjectures in such a way that the seller is better off offering prices $p_1, p_2, \ldots$ that maximize the utility of seller s', rather than surprising the buyer by offering prices not along the equilibrium path of seller s'. Finally, in equilibrium the buyer b_n, who is indifferent between accepting or rejecting p_n will strictly prefer to accept p_{n+1} rather than wait for p_{n+2}. Thus, to determine b_n it is sufficient to equate what the buyer b_n gets by accepting p_n and what he gets if he waits one period and accepts p_{n+1}.

Three cases are possible depending on the position of the seller's cost s relative to the support $[\underline{b}, \bar{b}]$ of the buyer's valuation. When s (possibly negative) is much less than $\underline{b}$, then the seller will offer $p = \underline{b}$ initially, so as to be sure to reach agreement immediately and avoid substantial costs of delay.[7] As s increases towards $\underline{b}$, at some point, say $s = d_2$, the seller is indifferent between offering $p = \underline{b}$ in the second period or waiting until the third period to offer $p = \underline{b}$. It is at this point that the incentive compatibility constraint becomes binding: if the seller's cost is less than d_2, then her behavior in subsequent rounds is not a function of s, so the buyer's behavior $\hat{b}$ does not depend on s, which implies that the seller has no

incentive to deceive the buyer into believing she has some cost $s' \neq s$; whereas, if the seller's cost is greater than d_2, then her second offer does depend on s, so the seller does have an incentive to fool the buyer. Finally, if $s > \underline{b}$ then the seller never offers the price $p = \underline{b}$, so that the bargaining could potentially continue indefinitely. Thus, the following three cases are possible:

(1) $s < d_2$: the incentive compatibility constraint is not binding, because the seller offers $p = \underline{b}$ in the first or second period so that the buyer's behavior does not depend on the seller's cost.

(2) $s > b$: bargaining may continue indefinitely.

(3) $d_2 < s < \underline{b}$: bargaining ends after a finite number of periods for all potential buyers, and the incentive compatibility constraint is binding.

<u>Case 1</u> $(s < d_2)$. The first case is handled easily. Suppose $s < d_2$ and that the second-period offer is $\underline{b}$. Then the seller's problem is

$$\max_p \; (p - s)(\bar{b} - \hat{b}) + \delta_s(\underline{b} - s)(\hat{b} - \underline{b})$$

$$\text{such that} \quad \hat{b} - p = \delta_b(\hat{b} - \underline{b})$$

Performing the optimization yields

$$p(s) = \max\left\{\underline{b}, \frac{1}{2}\left[\bar{b} - \delta_b(\bar{b} - \underline{b}) + s(1 - \delta_s) + \delta_s\underline{b}\right]\right\} \quad \text{and} \quad \hat{b}(p) = \frac{p - \delta_b\underline{b}}{1 - \delta_b} \; .$$

To compute the seller's cost d_2 at which the incentive compatibility constraint becomes binding, we simply equate the seller's utility if she offers $\underline{b}$ in the second period with her utility if she waits until the third period before offering $\underline{b}$:

$$u_2(s,\bar{b}) = u_3(s,\bar{b})$$

where $u_n(s,b_{n-1}) = 1/(b_{n-1} - \underline{b})[\alpha_n b_n^2 + (\beta_n s + \gamma_n)b_n + \rho_n s^2 + \sigma_n s + \tau_n]$. Thus, we wish to find s such that

$$\alpha\bar{b}^2 + (\beta x + \gamma)\bar{b} + \rho s^2 + \sigma s + \tau = 0$$

where $\alpha = \alpha_3 - \alpha_2$, $\beta = \beta_3 - \beta_2$, $\gamma = \gamma_3 - \gamma_2$, $\rho = \rho_3 - \rho_2$, $\sigma = \sigma_3 - \sigma_2$, and $\tau = \tau_3 - \tau_2$. Solving for s yields

$$s = x + \sqrt{x^2 - y}$$

where $x = -(\beta\bar{b} + \sigma)/2\rho$ and $y = (\alpha\bar{b}^2 + \gamma\bar{b} + \tau)/\rho$.

<u>Case 2</u> ($s > \underline{b}$). Now consider the case in which the incentive compatibility constraint is binding and $s > \underline{b}$. Suppose seller s chooses to pretend to be the seller s' by offering the price p. Then her expected payoff is given by

$$u_s(s'p) = \frac{1}{\bar{b} - \underline{b}}[(\bar{b} - \hat{b})(p - s) + \sum_{n=1}^{\infty} \delta_s^n(b_{n-1} - b_n)(p_n - s)] \qquad (U)$$

where the future prices and indifference valuations are

$$p_n = c(\hat{b} - s')d^{n-1} + s'$$

$$b_n = (\hat{b} - s')d^n + s'$$

and

$$\hat{b}(p) = \frac{p - \delta_b(1 - c)s'(p)}{1 - \delta_b + \delta_b c}$$

Thus,

$$b_{n-1} - b_n = (\hat{b} - s')(1 - d)d^{n-1}$$

$$p_n - s = c(\hat{b} - s')d^{n-1} + s' - s$$

so

$$(b_{n-1} - b_n)(p_n - s) = (\hat{b} - s')(1 - d)$$

$$[c(\hat{b} - s')(d^2)^{n-1} + (s' - s)d^{n-1}] \quad .$$

Substituting into (U) and performing the summation yields

$$u_s(s',p) = \frac{1}{\bar{b} - \underline{b}} [(\bar{b} - \hat{b})(p - s) + \sigma_s(\hat{b} - s')$$

$$(1 - d)[c(\hat{b} - s') \frac{1}{1 - \delta_s d^2} + (s' - s) \frac{1}{1 - \delta_s d}]]$$

It can be shown that $(1 - d)/(1 - \delta_s d^2) = 1/2$ and $(1 - d)/(1 - \delta_s d) = d$, so that

$$u_s(s',p) = \frac{1}{\bar{b} - \underline{b}}[(\bar{b} - \hat{b})(p - s)$$

$$+ \frac{1}{2} \delta_s c(\hat{b} - s')^2 + \delta_s d(\hat{b} - s') (s' - s)]$$

Taking the derivative of $u_s(s',p)$ with respect to p yields the first-order condition

$$\bar{b} - \hat{b} - \hat{b}_p(p - s) + \delta_s c(\hat{b} - s')(\hat{b}_p - s'_p)$$

$$+ \delta_s d[(\hat{b}_p - s'_p)(s' - s) + s'_p(\hat{b} - s')] = 0$$

where $\hat{b}_p$ is the derivative of $\hat{b}$ with respect to p and $s'_p = (ds'(p))/dp = ds/dp$. Making the substitutions $s' = s$ (implied by incentive compatibility), $s'_p = (ds/dp), \hat{b} = (1/v)(p - s) + s$, and $b_p = (1/v)(1 - (ds/dp) + ds/dp$ where $v = 1 - \delta_b(1 - c)$ yields the first-order differential equation

$$(\bar{b} - s)v^2 \frac{dp}{ds} - (p - s)[2v \frac{dp}{ds} - v + v^2 - \delta_s(c(\frac{dp}{ds} - 1) + vd)] = 0$$

which in differential form becomes

$$(\alpha p + \beta s + \gamma)dp + \sigma(p - s)ds = 0$$

where

$$v = 1 - \delta_b(1 - c)$$

$$\alpha = -2v + \delta_s c$$

$$\beta = v(2 - v) - \delta_s c$$

$$\gamma = \bar{b}v^2$$

$$\sigma = v(1 - v) + \delta_s(vd - c)$$

This differential equation is then solved using the procedure described in the appendix to yield $p(s)$ given the initial condition $p_o = p(\underline{s})$, where p_o is determined to maximize the payoff of seller $\underline{s}$ (the incentive compatibility constraint is not binding for the lowest-cost seller). The differential equation implies that each seller s prefers offering $p(s)$ than pretending to be any seller $s' \neq s$ by offering $p(s')$.

Case 3 $(d_2 \leq s < \underline{b})$. Finally suppose seller s chooses to pretend to be seller s' by offering the price p and ending negotiations with the price $\underline{b}$ after $n + 1$ periods. Then her expected payoff is given by

$$u_s(s',p) = \frac{1}{\bar{b} - \underline{b}} \left[(\bar{b} - \hat{b})(p - s) + \sum_{i=1}^{n} \delta_s^i (b_{i-1} - b_i)(p_i - s)\right]$$

where

$$b_i = x_i b_{i-1} + y_i s' + z_i$$

$$p_i = c_i b_{i-1} + d_i s' + e_i$$

and

$$\hat{b}(\hat{s},p) = \frac{p - \delta_b(d_n \hat{s} + e_n)}{1 - \delta_b(1 - c_n)} \quad .$$

Performing the optimization of $u_s(s',p)$ with respect to p subject to sequential rationality and incentive compatibility results in a first-order differential equation analogous to that found under Case 2.

Again, solution of the differential equation assures that seller s prefers offering $p(s)$ than $p(s')$ for all $s' \neq s$.

The analysis thus far has established the best-response strategies of the buyer and the seller when faced with the hypothesized equilibrium strategies. To assure that these best-response strategies do indeed form an equilibrium, one must verify that no player is better off deviating from these equilibrium strategies. How well a player can do by deviating will depend on the beliefs an opponent forms when faced with non-equililbrium behavior. Thus to determine that the seller is better off playing the equilibrium than deviating, I must posit the conjectures a buyer makes when faced with an offer off the equilibrium path.

One has a great deal of freedom in choosing conjectures that support an equilibrium. Since every seller wishes to be thought to have high costs, the conjecture most apt to support an equilibrium is "if an offer p is not an equilibrium offer, then $s = \underline{s}$ with probability one." However, such an extreme conjecture hardly seems plausible. The conjecture should be based on reasonable inferences a buyer might make when faced with an initial offer off the equilibrium path. To determine what constitutes "reasonable inferences" in particular applications, it is helpful to look at the equilibrium strategies and have the non-equilibrium beliefs be in line with the equilibrium beliefs. In the hypothesized equilibrium discussed here, higher prices signal higher costs. Thus, it is reasonable for the non-equilibrium conjectures to satisfy this monotonicity as well.

Describing conjectures that support an equilibrium is a complicated task in the infinite-horizon model, due to the many possibilities for deviant behavior the multiple periods afford. A seller could pretend to be someone else for a few periods and then start acting like himself or even a third type of seller; the possibilities for non-equilibrium behavior are practically limitless. However, one can describe a reasonably simple and intuitive set of conjectures that supports the equilibrium described here, in which the seller reveals completely her information over time.

One of the difficulties in establishing conjectures is the discontinuities in the equilibrium price schedule that arise when the seller's cost s is less than the valuation of buyer $\underline{b}$ and so the seller opts to end the bargaining after a finite number of periods. In particular, a discontinuity in the price schedule occurs at points d_n where seller d_n is indifferent between bargaining n periods and offering the price $\underline{p}(d_n) = \underline{\lim}_{s \to d_n} p(s)$ or bargaining $n + 1$ periods and offering the price $\bar{p}(d_n) = \overline{\lim}_{s \to d_n} p(s)$. Suppose the buyer observes the price $p \in (\underline{p}(d_n), \bar{p}(d_n))$, then I will assume that the buyer will believe $s = d_n$ with probability one and that the bargaining will continue for up to n periods.

Assuming that the discontinuity problem is resolved as described above, I can now define conjectures inductively that will support an equilibrium. In the posited conjectures, the buyer's beliefs will be of the same form whether he has observed behavior on or off the equilibrium path. Namely, at each stage of the game, the buyer either believes with

probability one that the seller has cost s or he believes that the seller's cost is uniformly distributed on the interval $[\hat{s},\bar{s}]$.

The buyer is assumed to have the following conjectures. Just before the nth period offer, the buyer believes that either $s = s_n < \hat{s}_n$ with probability one or that s is uniformly distributed on $[\hat{s}_n,\bar{s}]$. After the nth period offer p_n, the buyer revises his probabilities as follows:

(1) If the buyer believes $s \sim U[\hat{s}_n,\bar{s}]$ and $p_n \geq p_n(\hat{s}_{n+1})$, then $s \sim U[\hat{s}_{n+1},\bar{s}]$.

(2) If the buyer believes $s \sim U[\hat{s}_n,\bar{s}]$ and $p_n < p_n(\hat{s}_{n+1})$, then $s = p_n^{-1}(p_n)$.

(3) If the buyer believes $s = s_n$ and $p_n \geq p_n(\hat{s}_{n+1})$, then $s = s_n$.

(4) If the buyer believes $s = s_n$ and $p_n < p_n(\hat{s}_{n+1})$, then $s = \min\{s_n, p_n^{-1}(p_n)\}$.

This set of conjectures has two features tht make it especially desirable. First, it agrees with the notion that higher offers signal higher costs. Second, it yields beliefs off the equilibrium path that are similar to the equilibrium beliefs. Thus, a buyer's behavior changes continuously with changes in the price offered, whether or not the infinitesimal changes in price result in behavior off the equilibrium path.

The basic idea behind these conjectures is that a high-cost seller must be encouraged not to offer low prices initially, which are accepted

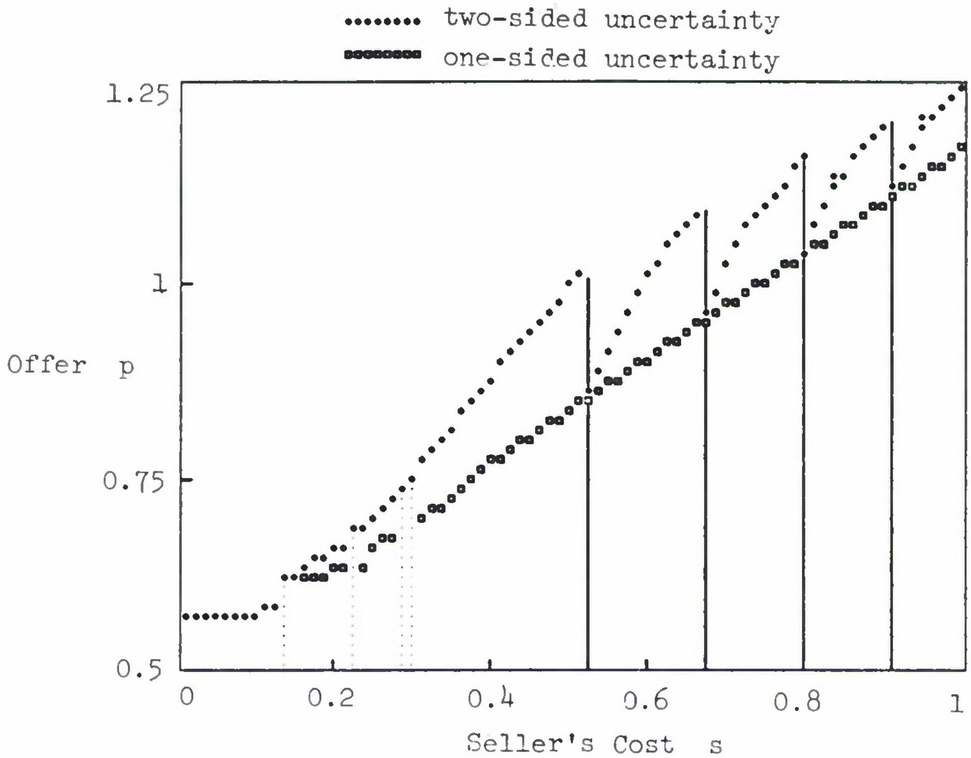
by some high-valuation buyers, and then revert to higher offers in later periods. Certainly if the buyer was naive, a high-cost seller would have an incentive to adopt such a strategy. The buyer's conjectures, however, make such an option unattractive, since the buyer when faced with non-equilibrium behavior always tends towards optimism: when the buyer is confused by non-equilibrium behavior, he assumes that the most optimistic information (that the seller has low costs) is valid. Showing that the posited conjectures support an equilibrium is a tedious task and is omitted.

Given specific values of δ_s, δ_b, $[\underline{s},\bar{s}]$, and $[\underline{b},\bar{b}]$, it is possible to compute an equilibrium by the following iterative procedure. First, compute the coefficients found in Section 4, which determine the offers and indifference valuations that occur after the seller's cost has been revealed. For the lowest-cost seller (seller $\underline{s}$), determine the price she should offer and her optimal maximum number of periods of bargaining. This is easily done, since the incentive compatibility constraint is nonbinding for seller $\underline{s}$. Gradually increase s from $\underline{s}$ by some small incremental step $\Delta s > 0$. At each iteration, find the price $p(s + \Delta s)$ that makes seller s indifferent between offering $p(s)$ and pretending to be seller $s + \Delta s$ by offering $p(s + \Delta s)$. As s increases, the seller will choose to extend the bargaining for more and more periods until the point $s = \underline{b}$ where the bargaining may continue indefinitely. For each s, the length of the bargaining is chosen to maximize the expected payoff of seller s. In addition, as s increases, there will eventually become a point $\hat{s}_1$ at

which no buyer will accept the price $p(\hat{s}_1)$. All sellers with costs $s > \hat{s}_1$ will offer an unacceptable price to signal that their costs are high. In the second round, prices for sellers $\underline{s}$ to $\hat{s}_1$ are easily determined since the incentive compatibility constraint is nonbinding for those sellers (they have already revealed their private information). For sellers $s > \hat{s}_1$, prices are determined as in the first period: the initial condition $p(\hat{s}_1)$ is easily determined, since the incentive compatibility constraint is nonbinding at this point; $p(s)$ then increases so that seller s is indifferent between offering $p(s)$ and $p(s + \Delta s)$, up until the point where no buyer accepts the second round offer $p(\hat{s}_2)$. This process is repeated until the equilibrium offers of all seller $s \in [\underline{s},\bar{s}]$ are determined.

As an example, consider the case when the seller's cost is uniformly distributed on $[0,1]$ and the buyer's valuation is uniformly distributed on $[1/2, 3/2]$ with $\delta_s = \delta_b = .75$, as shown in Figure 1. [Understanding this complicated figure is helpful to understanding the form of the equilibrium.] For $s < .14$, the bargaining ends after two periods and the incentive compatibility constraint is nonbinding. For $s > .14$, the incentive compatibility constraint is binding, which implies that the seller offers higher prices than she would had her costs been known to the buyer. This is as one would expect: the seller has an incentive to offer an inflated price to fool the buyer into believing her cost is greater than it actually is. The buyer, however, recognizes the seller's incentive to overstate her true cost, and so appropriately discounts the inflated offer. At $s = .52$ the seller

Figure 1. A separating equilibrium over time



offers a price so high that no buyer accepts. All sellers with higher costs wait until subsequent periods to reveal their private information. Thus, sellers $s \in [0,.52]$ reveal completely their information in the initial round of negotiations; seller $s \in [.52,.67]$ reveal their information in the second period; sellers $s \in [.67,.80]$ reveal their information in the third period, and so on. Every seller reveals her information by the end of the fifth period.

It is interesting to compare the seller's equilibrium price schedule with two-sided uncertainty in the infinite-horizon model with

her price schedule int he two-period model, as shown in Figure 2. First, notice that in the two-period model, the seller is able to reveal completely her information in the first round regardless of her cost; whereas, in the infinite-horizon model some sellers take up to five rounds to reveal their information. This is because when the seller only makes two offers, the seller's incentive to deceive in the initial period is reduced. Second, the seller offers higher prices in the two-period game than in the infinite-horizon game, since she is able to commit to ending the bargaining with a "take it or leave it" offer in the second period. In the infinite-horizon model, no such commitment is possible, so the seller is forced to offer lower prices.

6. Effect of Information, Uncertainty, and Time on Bargaining Behavior

In this section, I examine how information, uncertainty and time influence the bargaining behavior of rational agents. Throughout the section, comparisons are made between the infinite-horizon bargaining model with two-sided uncertainty and three other models:

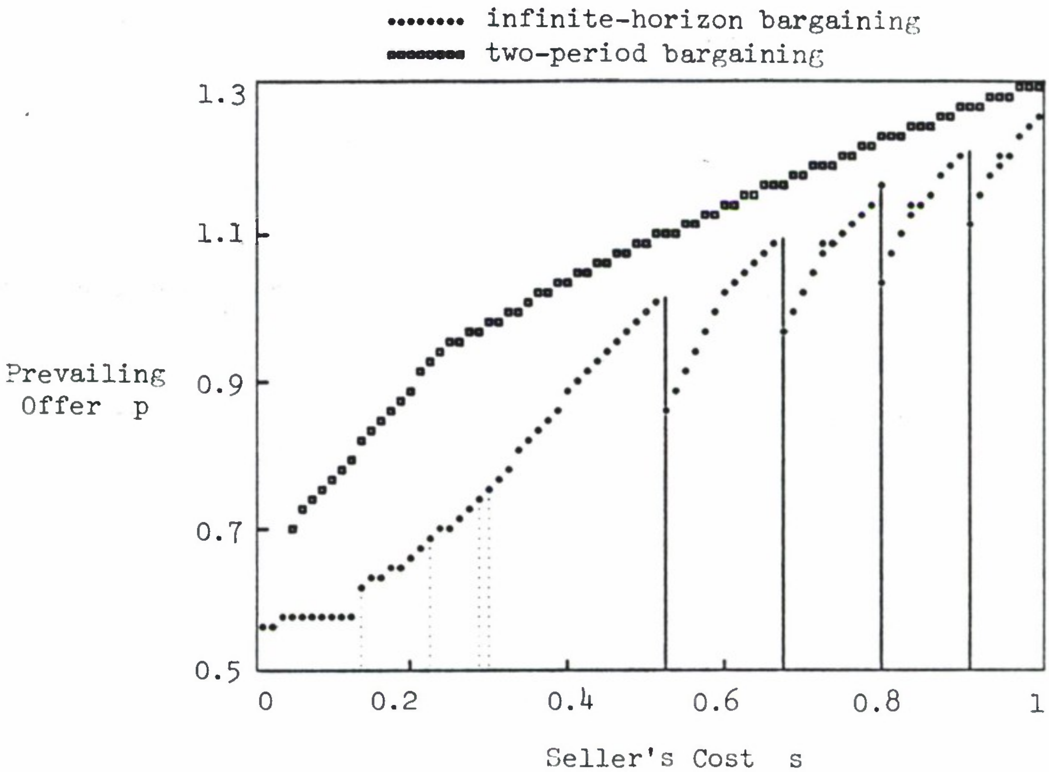
(1) the infinite-horizon bargaining model with one-sided uncertaity (the seller's cost known),

(2) the two-period model with two-sided uncertainty analyzed in Cramton [1983a], and

(3) the simultaneous-offer model studied by Chatterjee and Samuelson [1983].

Comparisons between the infinite-horizon models with two - and one-sided uncertainty shed light on how the bargainers cope with the additional

Figuere 2. Comparison between the two-period and infinite-horizon models.



uncertainty; whereas, comparisons between the infinite-horizon model and the two-period model suggest how the bargaining outcome changes when the players are able to commit to a shorter bargaining horizon. The simultaneous-offer model is of interest because it is the most efficient static bargaining mechanism (ex ante) that satisfies incentive compatibility and individual rationality for the class of examples considered here (Myerson and Satterthwaite [1983]). It, however, does not address the time dimension and violates a broad interpretation of sequential rationality: negotiations may end in a state of disagreement in which

it is common knowledge that there are substantial gains from trade.

Any comparisons must be made with reservation, due to the presence of multiple equilibria and the consideration of only uniform distributions; however, the separating equilibria found in each of the models seem quite reasonable and are unique among separating equilibria. Most of the results are sufficiently robust that they would not be affected by an alternative choice of equilibria or different distributions. A further complication in making comparisons is that efficiency is not well defined in settings of incomplete information, as pointed out by Holmstrom and Myerson [1981]. Does one make comparisons before the players know their private information, after they know their private information, or after all information is revealed? I will focus on ex ante efficiency, defined to be the ratio of the players' ex ante expected utilities and the expected gains from trade. Thus I integrate over the player's types before I divide by the expected gains from trade. Ex ante efficiency is the efficiency measure an uninformed social planner would use in deciding among bargaining mechanisms.

Information. I begin by exploring how the information available to the agents affects bargaining efficiency and the distribution of the gains from trade. With complete information, the bargaining is efficient - trade occurs without delay if and only if the seller's cost is less than or equal to the buyer's valuation. The split of the gains from trade depends on who is making the offers: when the seller makes all offers, she gets all the gains from trade; when the buyer makes all the offers, he gets all the gains; and when the players alternate

offers, the gains from trade are split with $(1 - \delta_b)/(1 - \delta_b\delta_s)$ going to the seller (assuming she makes the initial offer). In the presence of complete information, the offeror is at a great advantage, since the offeror can make an offer that an opponent is just willing to accept, thus extracting all the consumer surplus.

With uncertainty about an opponent's reservation price, the bargaining outcome is no longer efficient and the offeror is in a less dominant position. For example, with the seller's cost uniformly distributed on $[0,1]$ and the buyer's valuation uniformly distributed on $[1/2,3/2]$ with $\delta_b = \delta_s = .8$, so that the players have equal costs of delay and there is some overlap in their distributions, then the ex ante efficiencies and the proportion of the gains going to the seller are as shown below for the various bargaining settings:

Model	Ex ante Efficiency	Allocation (% to seller)
infinite-horizon with two-sided uncertainty	.75	.43
infinite-horizon with one-sided uncertainty	.90	.42
two-period with two-sided uncertainty	.69	.64
simultaneous offers	.90	.50

Several observations may be gleaned from the data. First, the inefficiencies causes by incomplete informatin are significant: one-quarter of the gains from trade are lost in the infinite-horizon bargaining game and one-tenth of the gains are lost when the players can commit to the simultaneous-offer game. Efficiency is reduced by 15% when the seller's cost is unknown, making both the seller and the buyer worse off.

Although the simultaneous-offer game is more efficient ex ante than the infinite-horizon game, not every trader prefers the simultaneous-offer game. In fact, when both the sellers and the buyers are uniformly distributed on $[0,1]$, one-quarter of the players in the simultaneous-offer game receive nothing in equilibrium. In the infinite-horizon game, almost every seller expects strictly positive gains from trade. Thus, if the players were to choose which game to play, a naive player's choice of game would reveal information to the opponent. Moreover, implicit in the simultaneous-offer game is the requirement that the bargainers will end negotiations after the first offer. The problem with this requirement is that the simultaneous-offer game ends with positive probability in a state of disagreement in which it is common knowledge that substantial gains from trade exist. In the infinite-horizon game, the bargainers are unable to commit to walking away from positive gains from trade.

Perhaps somewhat surprisingly, the offeror is at a disadvantage in the infinite-horizon game, receiving only 43% of the gains. However, when the bargaining is limited to two periods, then the offeror is at an advantage, receiving 64% of the gains. This is because in the infinite-horizon game the seller is unable to commit to high prices in the future, as she can in the two-period model. It seems in this case that the structure of the game forces the seller to reveal more information than she would like to: she would prefer to let the buyer make the offers. The result that the offeror is at a disadvantage in the infinite-horizon bargaining game depends critically on the degree of

uncertainty as discussed below.

Uncertainty. In addition to information, the degree of uncertainty the players have about each other's reservation prices will affect bargaining efficiency. Here uncertainty is measured as the degree of overlap in the distributions of buyers and sellers: if the distributions are separated by a large amount so that both the buyer and the seller are confident of large gains from trade, the uncertainty is slight; whereas, if the distributions completely overlap, the uncertainty is great.[8/] Thus, I test the sensitivity of the bargaining outcome on uncertainty by changing the support of the distribution of one of the players (in this case the buyer's distribution).

A plot of efficiency and the allocation of gains is shown in Figure 3 when the support of the buyers varies from [0,1] to [2,3] with the sellers on [0,1] and $\delta_b = \delta_s = .8$. Efficiency increases monotonically in both models as the uncertainty is reduced. The difference in efficiency between the infinite-horizon model and the simultaneous-offer model increases as the gains from trade become more uncertain. Thus, the value of commitment increases with uncertainty.

In the simultaneous-offer game, the gains from trade are split evenly regardless of the degree of uncertainty. However, in the infinite-horizon model, the seller's proportion of the gains decreases monotonically as the uncertainty increases. When the uncertainty is greatest (complete overlap of the reservation price distributions), the seller gets only 35% of the gains; whereas, when the gains from trade are known to be large $((1/2)(\underline{b} + \bar{b}) = 2.5)$, the seller gets 75% of the

Figure 3

Effect of uncertainty on the Bargaining outcome.

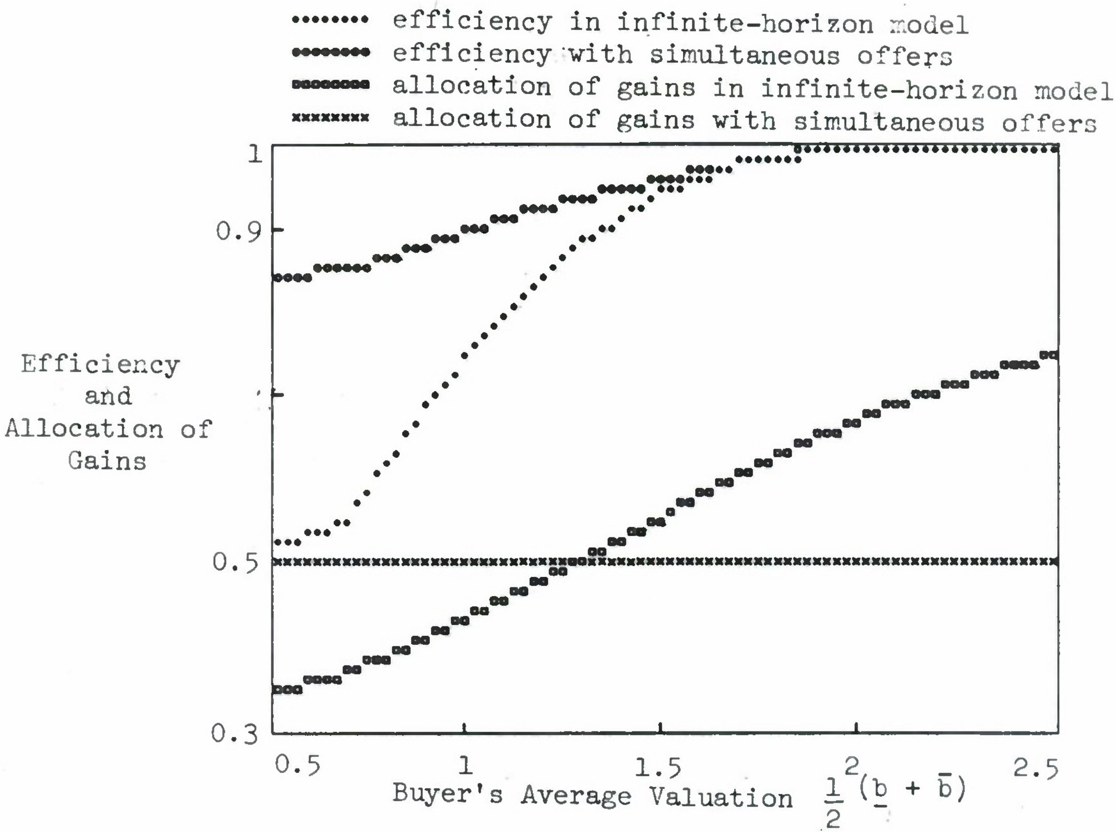
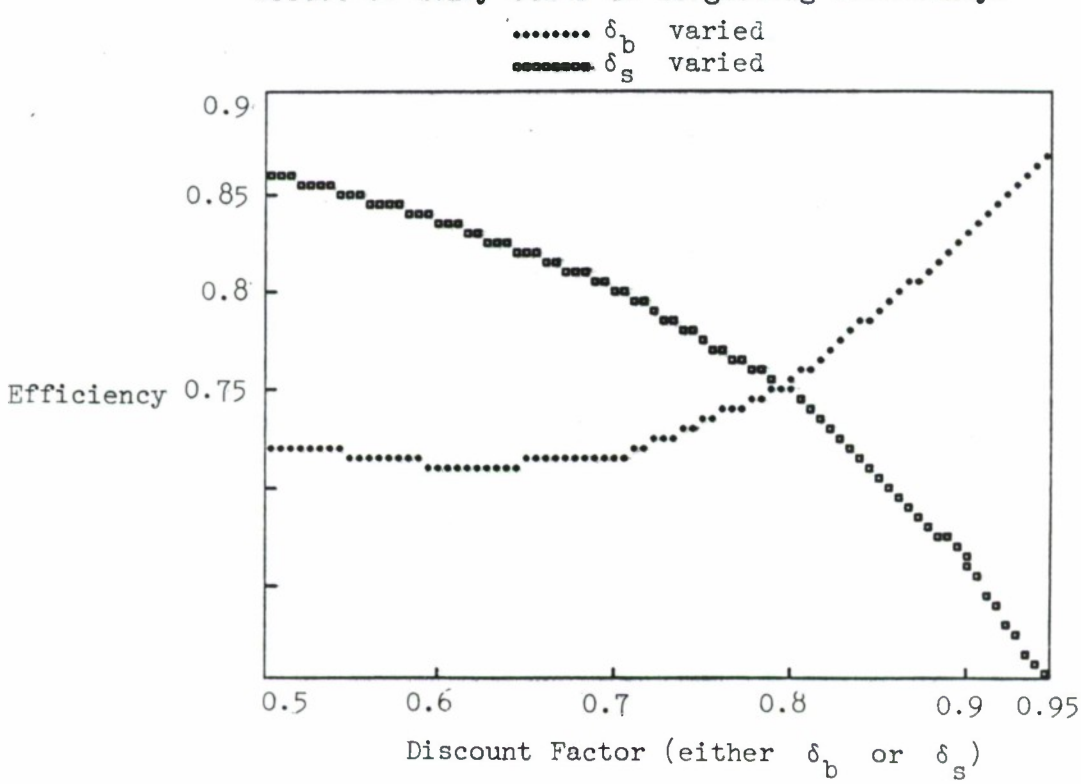


Figure 4

Effect of delay costs on bargaining efficiency.



gains. This result is in agreement with the common wisdom among negotiators that says that when the gains from trade are highly uncertain it is best to let your opponent make the initial offer (and thus reveal valuable information about his reservation price), but when the gains from trade are known it is best to go first (Below and Moulton [1981], pages 104-106).

Impatience. Finally, I consider the effect of varying the players'costs of delay on the bargaining outcome. A plot of how relative changes in the players' delay cost influence the bargaining efficiency is shown in Figure 4. The plot is made assuming the seller's cost is uniformly distributed on $[0,1]$ and the buyer's valuation is uniformly distributed on $[1/2,3/2]$ with one of the player's discount factors held fixed at .8 and the other varied from .5 to .95. In the infinite-horizon model, the effect of delay costs on efficiency is nearly monotone. As the buyer's delay costs increase the bargaining becomes less efficient, since the seller, who is controlling the bargaining, is relatively more willing to wait, so she makes higher offers and the buyer suffers costly delay. In contrast, as the seller's delay costs increase, the bargaining becomes more efficient, since with high delay costs the seller offers lower prices, which are accepted earlier. The infinite-horizon model, then, is most efficient when the offeror's delay costs are relatively large. In fact, as $\delta_s \to 0$ and $\delta_b \to 1$, the seller loses all of her bargaining power and efficiency goes to 1. Thus, there exist discount factors for which the infinite-horizon bargaining game is more efficient than the ex ante efficient static

mechanism. This reult is quite general and is derived in Cramton [1983b].

The reason the sequential game may be more efficient than the most efficient static game is that, in the sequential game, time may introduce an additional asymmetry into the problem (the players may have different delay costs), which can influence the efficiency of the bargaining outcome. Thus, although incomplete information in sequential games typically leads to inefficient outcomes, there exists a range of parameters for which the sequential game is more efficient ex ante than the ex ante efficient static game. As the seller's and buyer's discount factor go to 0 and 1 respectively, the seller loses all her bargaining power and is forced to make offers arbitrarily close to her reservation price. Static mechanisms require that the expected payment between players and the probability of agreement are identical for any realization of the bargaining mechanism. This, however, is not true in a sequential game: the discounted expected payment and the discounted probability of agreement may be different due to differences in the players; discount factors.

Allocation of the gains from trade changes monotonically with changes in the players' delay costs, as shown in Figure 5. The seller's proportion of the gains from trade increases monotonically as δ_s decreases. As one would expect, patience is a virtue.

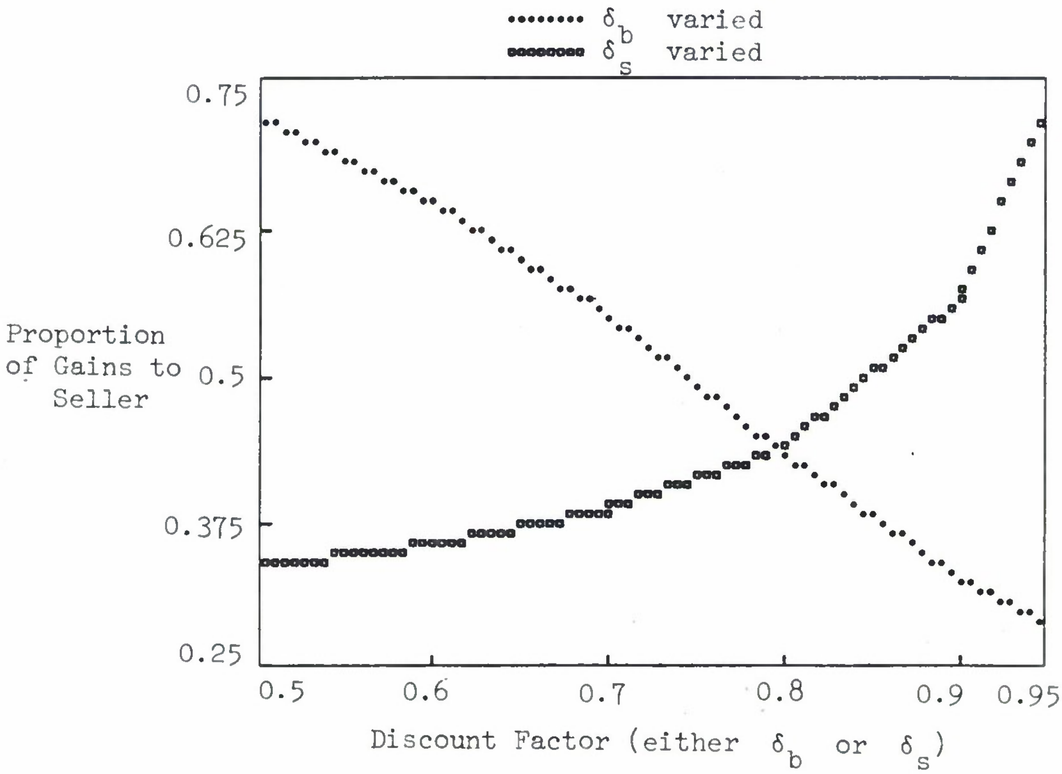
Next I consider absolute changes in the players' delay costs by varying both bargainers discount factors simultaneously (assuming $\delta_s = \delta_b$). As shown in Figure 6, the effect of absolute changes in delay costs on efficiency depends on the degree of uncertainty. When there is

a great deal of uncertainty $(b \sim U[0,1])$, efficiency increases monotonically with delay costs. When there is moderate uncertainty $(b \sim U[1/2, 3/2])$, then efficiency is roughly constant as delay costs are varied from .5 to .95. Finally when uncertainty is slight, efficiency increases as delay costs decrease. Intuitively, when uncertainty is great, large delay costs improve efficiency by forcing the bargainers to come to an early agreement; whereas, when uncertainty is slight, an early agreement is reached regardless of the magnitude of delay costs and so higher delay costs reduce efficiency by increasing costs of agreement.

Regardless of the degree of uncertainty, increasing delay costs benefits the seller and hurts the buyer (see Figure 7), because with higher delay costs the seller can force the buyer to accept higher

Figure 5

Effect of delay costs on the allocation of gains



prices. Thus, one would expect the seller to choose to lengthen the time between offers; whereas, the buyer would prefer offers to be made in rapid succession.

7. Conclusion

In any realistic bargaining setting the issues of information, timing, and commitment are of crucial importance. Informational differences among agents often lead to inefficient bargaining outcomes and deadlocked negotiations. Time pressures tend to force an early resolution of the bargaining conflict. And the ability of agents to commit to particular strategies often determines how the gains from trade are divided among the agents. In this paper, I have presented an

Figure 6.

Effect of absolute changes in delay costs on efficiency

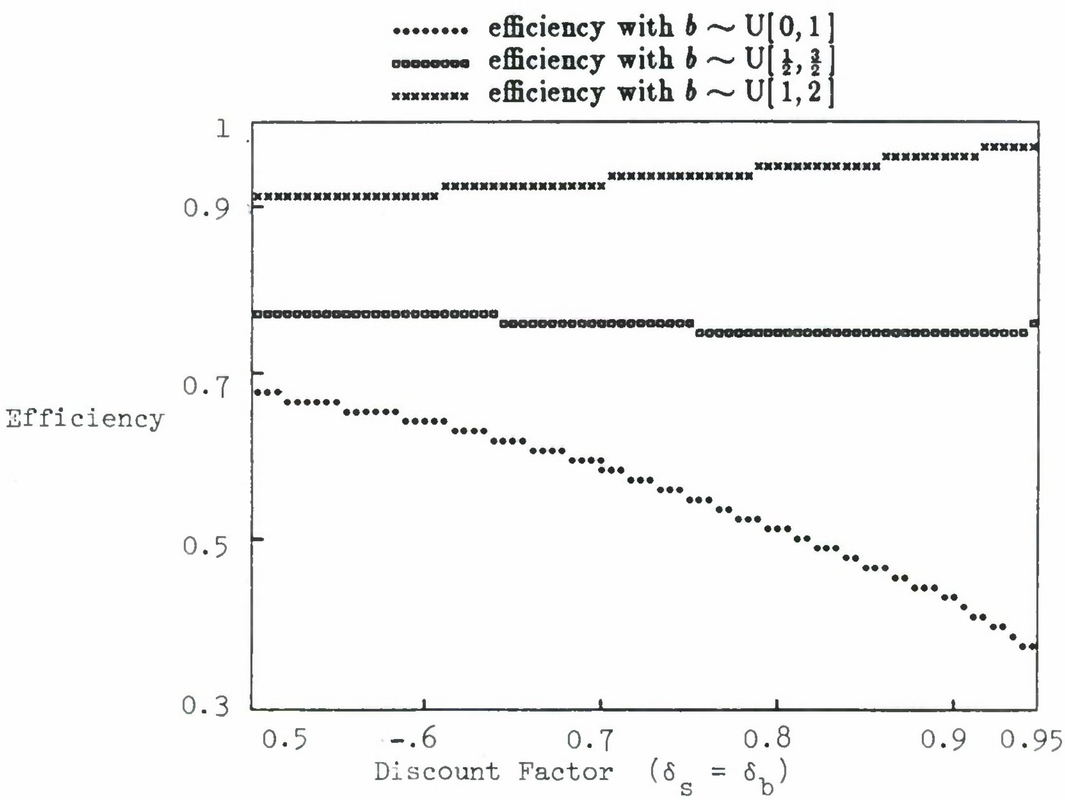
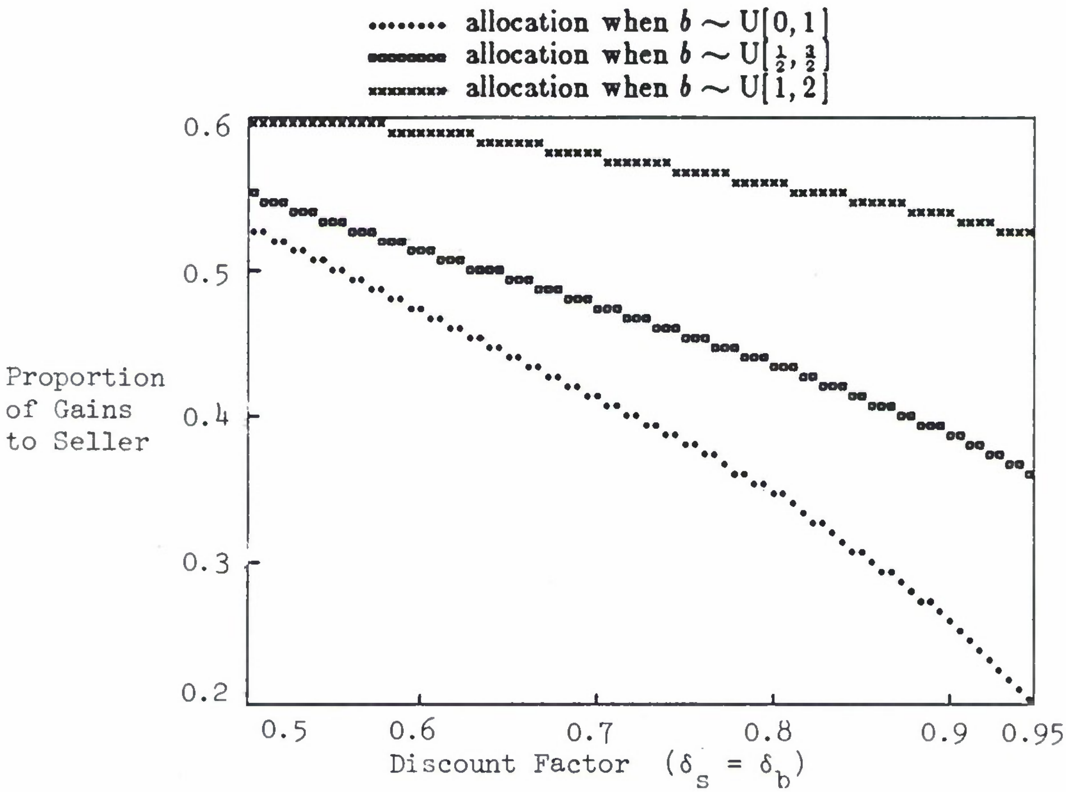


Figure 7.

Effect of abosolute changes in delay costs on allocation of gains



infinite-horizon bargaining model that explicitly considers information and timing in bargaining settings in which the players are unable to precommit to particular strategies. Rational behavior on the part of the agents has been characterized for a class of distributions under various information structures.

The results of the example are intuitively appealing. When the agents have complete information an efficient agreement is immediately reached. When only the buyer has complete information, trade frequently occurs only after costly delay. Moreover, the buyer benefits from his

superior information. When both the buyer and the seller are unsure of the other's preferences, the outcome is even less efficient, due to the seller's incentive to deceive the buyer. How much information the seller reveals in each round of negotiations depends on the players' costs of delay: when delay costs are high, much information is revealed each period; whereas, when the costs of delay are small, less information is revealed. Due to the seller's incentive to lie about her costs, the seller must offer higher prices than she would had the buyer known her cost. These higher prices lead to significant inefficiencies: roughly 25% of the potential gains from trade are lost in a typical example.

Both the degree of uncertainty and the costs of delay affect the bargaining outocme. Uncertanty has a detrimental effect on bargaining efficiency - the more uncertainty present, the less efficient the bargaining outcome. In constrast, costs of delay, as modeled by the players' discount factors, tend to have a positive effect on bargaining efficiency - higher costs of delay tend to increase efficiency. The effect, however, is sometimes ambiguous: higher costs of delay will increase the probability of an early agreement, but reduce the benefit of a delayed agreement.

An important feature of the bargaining game presented here, is that it makes no assumptions about the bargainers' ability to commit to future strategies: the players continue to negotiate so long as they expect positive gains from continuing. Implicit in some bargaining mechanisms is the assumption that the bargainers are able to commit to

walking away from the negotiating table, even when it is common knowledge that the gains from trade are positive. The simultaneous-offer game is an example of such a mechanism. Although this game is more efficient ex ante, it ends with positive probability in a state in which both bargainers know that gains from trade exist (since their respective reservation prices have been revealed), and yet they are forced to walk away from the bargaining table. Thus, the bargaining game implicitly assumes that the players are able to commit to walking away without trading, after it has been revealed that substantial gains from trade exist. This point is addressed in greater detail in Cramton [1983b].

It is somewhat disappointing that the bargaining game analyzed here is not more efficient. When the players' reservation prices are uniformly distributed on $[0,1]$ and their discount factors are equal, this game results in at least 32% of the gains from trade being lost, as opposed to a 16% loss if the ex ante efficient bargaining mechanism is adopted. An important question to answer in future research is can we find a strategic game that implements (or comes close to implementing) the ex ante efficient perfect bargaining mechanism over a wide range of bargaining situations? Perhaps a better candidate for a strategic bargaining game that is nearly ex ante efficient is the game in which the bargainers alternate offers. This game was analyzed by Rubinstein [1982] in a setting of complete information, but an analysis with incomplete information has yet to be done. Of particular interest is the alternating-offer game as the time between offers goes to zero, for

this strategic game represents a very general bargaining rule: at any time a bargainer may make a new offer or accept the most recent offer of his opponent. It would be a pleasant surprise if such a reasonable bargaining game was ex ante efficient over a variety of circumstances.

The model presented here is far from being a complete description of most bargaining situations. Several restrictive assumptions have been made in order to make the analysis manageable. First, the agents have been assumed to be risk neutral, but in many bargaining situations the agents' willingness to take risks is an important bargaining factor. Second, I have restricted attention to rational agents who can calculate (at no cost) their optimal strategies. Certainly, few agents are so consistent and calculating. With less than rational agents, an agent's capacity to mislead his opponent becomes an important variable in determining how the gains from trade are divided. Finally, I consider only a bargaining setting in which agents are unable to commit to particular strategies. In many real-life situations, bargainers will often have or create a means of commitment. For example, an agent anticipating that he will be faced with a similar situation in the future may wish to establish a reputation for toughness, as in the case of a manager refusing an employee's request for a pay raise on the grounds that he would have to do the same for everyone else.

Two weaknesses of the noncooperative game-theoretic approach are worth mentioning. First, even in the case of incomplete information, the information requirements of the players are severe: some base of common knowledge must exist. Thus, although neither player need know

the other's reservation price, they must know (and know the other knows, and so on) each other's probabilistic beliefs about the other's reservation price. In practice, these beliefs are not common knowledge, so I am implicitly assuming that their beliefs are sufficiently aligned that they act as if their beliefs were common knowledge. A second weakness of the noncooperative approach is that a particular game structure must be specified. Why should the agents play the specified game? Who decides which game to play? These are important questions, which should be studied. Although it is unreasonable to assume that the agents are playing the exact game specified, one can often assume that the game being played is a close enough approximation to the specified game that useful insights into the agents' behavior can be gained.

Despite these limitations, many of which can be addressed in future research, I feel that the approach of modeling bargaining as a noncooperative sequential game has many merits over other approaches, such as cooperative game theory. Perhaps the most important advantage of the noncooperative approach is that it explicitly models the behavior of the bargainers and does not assume at the outset that an efficient solution will be reached. Cooperative bargaining, on the other hand, focuses entirely on the bargaining outcome and assumes that the bargaining will be efficient, contrary to the common occurrence of inefficient bargaining outcomes in practice.

Appendix

A.1. Equilibrium Strategies when the Seller's Cost is Known and $s < \underline{b}$

Theorem 2: For the n-stage game with $s < \underline{b}$ in which the seller ends the bargaining by offering $p_n = \underline{b}$, the seller's price $p_j(s,b_{j-1})$, her expected profit $u_j(s,b_{j-1})$, and the buyer's indifference valuation $b_j(s,b_{j-1})$ with i periods remaining and $j = n + 1 - i$ are given by

$$b_j(s,b_{j-1}) = x_j b_{j-1} + y_j s + z_j$$

$$p_j(s,b_{j-1}) = c_j b_{j-1} + d_j s + e_j$$

$$u_j(s,b_{j-1}) = \frac{1}{\bar{b} - \underline{b}} [\alpha_j b_{j-1}^2 + (\beta_j s + \gamma_j) b_{j-1} + \rho_j s^2 + \sigma_j s + \tau_j]$$

where $c_n = 0$, $d_n = 0$, $e_n = \underline{b}$, $\alpha_n = 0$, $\beta_n = -1$, $\gamma_n = \underline{b}$, $\rho_n = 0$, $\sigma_n = \underline{b}$, and $\tau_n = -\underline{b}^2$, and for $i > 1$

$$v_j = 1 - \delta_b + \delta_b c_{j+1}$$

$$w_j = 2(v_j - \delta_s \alpha_{j+1})$$

$$x_j = v_j / w_j$$

$$y_j = (1 + \delta_s \beta_{j+1} - \delta_b d_{j+1}) / w_j$$

$$z_j = (\delta_s \gamma_{j+1} - \delta_b e_{j+1}) / w_j$$

$$c_j = v_j^2 / w_j$$

$$d_j = [v_j(\delta_s\beta_{j+1} + \delta_b d_{j+1} + 1) - 2\delta_s\delta_b\alpha_{j+1}d_{j+1}]/w_j$$

$$e_j = [v_j(\delta_s\gamma_{j+1} + \delta_b e_{j+1}) - 2\delta_s\delta_b\alpha_{j+1}e_{j+1}]/w_j$$

$$\alpha_j = v_j x_j(1 - x_j) + \delta_s\alpha_{j+1}x_j^2$$

$$\beta_j = (1 - 2x_j)v_j y_j - (1 - x_j)(1 - \delta_b d_{j+1}) + \delta_s(2\alpha_{j+1}x_j y_j + \beta_{j+1}x_j)$$

$$\gamma_j = (1 - 2x_j)v_j z_j + (1 - x_j)\delta_b e_{j+1} + \delta_s(2\alpha_{j+1}x_j z_j + \gamma_{j+1}x_j)$$

$$f_j = 1 - \delta_b d_{j+1} + \delta_s\beta_{j+1} + y_j(\delta_s\alpha_{j+1} - v_j)$$

$$g_j = \delta_s\gamma_{j+1} - \delta_b e_{n-1} + z_j(\delta_s\alpha_{j+1} - v_j)$$

$$\rho_j = f_j y_j + \delta_s\rho_{j+1}$$

$$\sigma_j = f_j z_j + g_j y_i + \delta_s\sigma_{j+1}$$

$$\tau_j = g_j z_j + \delta_s\tau_{j+1}$$

<u>Proof</u>: By assumption, with one period remaining, $p_n = \underline{b}$ and $u_n = \underline{b} - s$.

With i periods remaining and $j = n + 1 - i$, the seller's expected profit is given by

$$u_j(s,n_{j-1}) = \max_p \frac{1}{b_{j-1} - \underline{b}}\,[(p - s)(b_{j-1} - b_j) + \delta_s(b_j - \underline{b})u_{j+1}(s,b_j)] \tag{5}$$
$$\text{such that } b_j - p = \delta_b(b_j - p_{j+1})$$

Assume by the induction hypothesis that

$$p_{j+1}(s,b_j) = c_{j+1}b_j + d_{j+1}s + e_{j+1}$$

$$u_{j+1}(s,b_j) = \frac{1}{b_j - \underline{b}}\left[\alpha_{j+1}b_j^2 + (\beta_{j+1}s + \gamma_{j+1})b_j + \rho_{j+1}s^2 + \sigma_{j+1}s + \tau_{j+1}\right]$$

So $p = (1 - \delta_b)b_j + \delta_b(c_{j+1}b_j + d_{j+1}s + e_{j+1})$, or

$$(6) \qquad p = (1 - \delta_b + \delta_b c_{j+1})b_j + \delta_b(d_{j+1}s + e_{j+1})$$

Dropping the $j + 1$ subscripts and substituting into (5), yields

$$(7) \qquad \begin{aligned} u_j(s,b_{j-1}) = \max_b \frac{1}{b_{j-1} - \underline{b}} &\left[(1 - \delta_b + \delta_b c)b + \delta_b(ds + e) - s\right](b_n - b) \\ &+ \delta_s\left[\alpha b^2 + (\beta s + \gamma)b + \rho s^2 + \sigma s + \tau\right] \end{aligned}$$

which has a unique maximum when

$$(8) \qquad b = \frac{(1 - \delta_b + \delta_b c)b_{j-1} - \delta_b(ds + e) + s + \delta_s(\beta s + \gamma)}{2(1 - \delta_b\delta_b + \delta_b c - \delta_s\alpha)}$$

Substituting (8) into (6) and (7), yields the derised expressions for $p_j(s,b_{j-1})$ and $u_j(s,b_{j-1})$. Q.E.D.

A.2. Differential Equation for a Separating Equilibrium

Here I solve the following differential equation, whch arises when determining a separating equilibrium

$$(a_1p + a_2s + a_3)dp + (b_1p + b_2s + b_3)ds = 0$$

where a_i and b_i, $i = 1,2,3$ are constants. The differential equation (DE) is solved in three steps.

1. Transform the variables to make the equation (DE) homogenous of degree one. Let $p = y + \bar{p}$ and $s = x + \bar{s}$, where $\bar{p}$ and $\bar{s}$ are chosen to solve

$$a_1 p + a_2 s + a_3 = 0$$

$$b_1 p + b_2 s + b_3 = 0$$

which implies

$$\bar{p} = \frac{a_2 b_3 - a_3 b_2}{a_1 b_2 - a_2 b_1} \quad ; \quad \bar{s} = \frac{a_3 b_1 - a_1 b_3}{a_1 b_2 - a_2 b_1}$$

Making the substitution

$$p = y + \frac{a_2 b_3 - a_3 b_2}{a_1 b_2 - a_2 b_1} \quad ; \quad dp = dy$$

$$s = x + \frac{a_3 b_1 - a_1 b_3}{a_1 b_2 - a_2 b_1} \quad ; \quad ds = dx$$

results in the homogeneous equation

$$(a_1 y + a_2 x)dy + (b_1 y + b_2 x)dx = 0 \qquad \text{(H)}$$

2. Transform the variables to make the equation (H) separable. Let $y = vx$ and $dy = vdx + xdv$ to get the separable equation

$$(a_1 v^2 + (a_2 + b_1)v + b_2)dx + (a_1 v + a_2)xdv = 0$$

or

$$\frac{dx}{x} + \frac{a_1 v + a_2}{a_1 v^2 + (a_2 - b_1)v + b_2} dv = 0 \qquad \text{(S)}$$

3. Integrate the separable differential equation (S). First factor the quadratic as follows:

$$a_1 v^2 + (a_2 + b_1)v + b_2 = a_1(v + f)(v + g)$$

where

$$f = \frac{1}{2a_1}(a_2 + b_1 + d)$$

$$g = \frac{1}{2a_1}(a_2 + b_1 - d)$$

$$d = \sqrt{(a_2 + b_1)^2 - 4a_1 b_2}$$

So

$$\frac{a_1 v + a_2}{a_1 v^2 + (a_2 + b_1)v + b_2} = \frac{v + a_2/a_1}{(v + f)(v + g)}$$

Now find s and t such that

$$s(v + f) + t(v + g) = v + a_2/a_1$$

which implies

$$s = \frac{1}{2d}(a_2 - b_1 + d) \qquad t = \frac{1}{2d}(b_1 - a_2 + d)$$

so

$$\frac{a_1 v + a_2}{a_1 v^2 + (a_2 + b_1)v + b_2} = \frac{s}{v + g} + \frac{t}{v + f} \quad .$$

Then integrating (S), yields $x(v + g)^s(v + f)^t = K'$, or

$$x^{2d}(v + g)^{a_2 - b_1 + d}(v + f)^{b_1 - a_2 + d} = K \ .$$

Substituting back to the original variables results in the primmitive:

$$(s - \bar{s})^{2d}(h + g)^{d+e}(h+f)^{d-e} = K \qquad \text{(P)}$$

where

$$\bar{s} = \frac{a_3 b_1 - a_1 b_3}{a_1 b_2 - a_2 b_1}$$

$$\bar{p} = \frac{a_2 b_3 - a_3 b_2}{a_1 b_2 - a_2 b_1}$$

$$d = \sqrt{(a_2 + b_1)^2 - 4a_1 b_2}$$

$$e = a_2 - b_1$$

$$f = \frac{1}{2a_1}(a_2 + b_1 + d)$$

$$g = \frac{1}{2a_1}(a_2 + b_1 - d)$$

$$h = \frac{p - \bar{p}}{s - \bar{s}}$$

and K is chosen such that $p(s_o) = p_o$.

The primitive equation (P) can easily be solved for p as a function of s for specific values of the constants a_i and b_i, $i = 1,2,3$.

Footnotes

1/ I have assumed arbitralily that the seller is female and the buyer is male

2/ In order to simplify notation, I will occasionally drop the time subscript. Naturally, the players' strategies depend on their current beliefs about their opponent' reservation price. These beliefs change over time so the functions p and b change over time as well

3/ For notational simplicity, I omit the functional dependence of the players' strategies on parameters of the model that are known and constant throughout the game, such as ${}_s$, ${}_b$, $\underline{b}$, $\bar{b}$, $\underline{s}$ and $\bar{s}$.

4/ This theorem appears in slightly modified form as Theorem 6 in Sobel and Takahashi [1983]. The difference between the two is that here the seller has some nonzero cost s to acquire the object, and the buyer's valuation is uniformly distributed on $[\underline{b},\bar{b}]$, rather than distributed on $[0,1]$ with the distribution b^m for $m > 0$.

5/ A necessary and sufficient condition for the strict concavity of u_j is ${}_b(1 - c_{j+1}) + (1/2)\ {}_s c_{j+1}$ 1. This is clearly satisfied, since ${}_s$, ${}_b$, and c_{j+1} are between 0 and 1.

6/ Actually, I need to show that the limit of the equilibrium strategies in the finite-horizon game converges to an equilibrium in the infinite-horizon game. Indeed this is the case for this game, as shown by Fudenberg and Levine [1981]. Moreover, Fudenberg, Levine, and Tirole [1983] show that this equilibrium is the unique equilibrium in the infinite-horizon game.

7/ An offer of $\underline{b}$ guarantees agreement because the seller makes all the offers, and thus is able to commit to never offering a price below $\underline{b}$. In an alternating-offer model, prices below $\underline{b}$ would have to be considered.

8/ I define increasing uncertainty as a shift to the left of the distribution of the gains from trade.

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